2001 AB 2 yes calculator

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

   (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

   (c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

   (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

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6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

   (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

   (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

SCORING
#2

(a) Difference quotient; e.g. $W'(12) \approx \frac{W(15)-W(12)}{15-12} = -\frac{1}{3}$ °C/day or $W'(12) \approx \frac{W(12)-W(9)}{12-9} = -\frac{2}{3}$ °C/day or $W'(12) \approx \frac{W(15)-W(9)}{15-9} = -\frac{1}{2}$ °C/day	$2: \begin{cases} 1: \text{difference quotient} \\ 1: \text{answer (with units)} \end{cases}$
(b) $\frac{3}{2}(20+2(31)+2(28)+2(24)+2(22)+21) = 376.5$ Average temperature $\approx \frac{1}{15}(376.5) = 25.1\,°\text{C}$	$2: \begin{cases} 1: \text{trapezoidal method} \\ 1: \text{answer} \end{cases}$
(c) $P'(12) = 10e^{-t/3} - \frac{10}{3}te^{-t/3}\Big\vert_{t=12}$ $= -30e^{-4} = -0.549$ °C/day This means that the temperature is decreasing at the rate of 0.549 °C/day when $t = 12$ days.	$2: \begin{cases} 1: P'(12) \text{ (with or without units)} \\ 1: \text{interpretation} \end{cases}$
(d) $\frac{1}{15}\int_0^{15}\left(20+10te^{-t/3}\right)dt = 25.757\,°\text{C}$	$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits and} \\ \quad \text{average value constant} \\ 1: \text{answer} \end{cases}$

Notes: the third method shown for Part A makes the most sense in that it is the most repeatable. Imagine, for instance, if they had asked for estimating $W'(13)$...you would use the points given on either side of it.

Part C should be done using “Math-8” in the calculator.

SCORING
#6

(a) $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}(6-2x) - 2y^2$

$= 2y^3(6-2x)^2 - 2y^2$

$\left.\frac{d^2y}{dx^2}\right|_{\left(3,\frac{1}{4}\right)} = 0 - 2\left(\frac{1}{4}\right)^2 = -\frac{1}{8}$

$3: \begin{cases} 2: \frac{d^2y}{dx^2} \\ \quad < -2 > \text{ product rule or chain rule error} \\ 1: \text{value at } \left(3, \frac{1}{4}\right) \end{cases}$

(b) $\frac{1}{y^2}\,dy = (6-2x)\,dx$

$-\frac{1}{y} = 6x - x^2 + C$

$-4 = 18 - 9 + C = 9 + C$

$C = -13$

$y = \frac{1}{x^2 - 6x + 13}$

$6: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivative of } dy \text{ term} \\ 1: \text{antiderivative of } dx \text{ term} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition } f(3) = \frac{1}{4} \\ 1: \text{solves for } y \end{cases}$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

Notes: Part A deals with "implicit differentiation." We did not do a whole lot with implicitly finding a second derivative, but it isn't too challenging with practice.

Part B: Notice how the key has you find the value of C before solving for y. Sometimes this is beneficial, but knowing when it can be also comes from practice. The fact that the output is a fraction (ie, $f(3) = \frac{1}{4}$) is a good indicator that it might be worth a shot.