

- $2(x^3 + 1)^1 (3x^2) = 6x^2(x^3 + 1)$ **[E]**
- $u = -4x$ $du = -4 dx$ Thus integral is $\frac{e^{-4x}}{-4}$ so this give $\frac{e^{-4}}{-4} - \frac{1}{-4} = \frac{1}{4} - \frac{e^{-4}}{4}$ **[D]**
- Horizontal asymptotes occur as the function gets very large or very small, thus
 $\lim_{x \rightarrow \infty} f(x) = 2$ **[E]**
- $\frac{(3x+2)(2) - (2x+3)(3)}{(3x+2)^2} = \frac{6x+4-6x-9}{(3x+2)^2} = \frac{-5}{(3x+2)^2}$ **[D]**
- Integral of $\sin x$ is $-\cos x$ so $-\cos\left(\frac{\pi}{4}\right) - (-\cos(0)) = -\frac{\sqrt{2}}{2} - (-1) = -\frac{\sqrt{2}}{2} + 1$ **[D]**
- Use principal of dominance and use largest power over largest power or
 $\lim_{x \rightarrow \infty} \frac{x^3}{4x^3} \rightarrow \frac{1}{4}$ **[C]**
- When a derivative is below the x-axis it is negative and thus $f(x)$ is decreasing and when a derivative is above the x-axis it is positive and thus $f(x)$ is increasing. From -2 to 0 the derivative is above the x-axis, so $f(x)$ is increasing. **[B]**
- $u = x^3$ $du = 3x^2 dx$ So $\frac{du}{3} = x^2 dx$ The integral of $\int \left(\frac{\cos u}{3}\right) du = \frac{1}{3} \sin u + C$. So substituting in for u gives $\frac{1}{3} \sin x^3 + C$ **[B]**
- $f'(x) = \frac{1}{x+4+e^{-3x}} \cdot (1-3e^{-3x})$ so $f'(0) = \frac{1}{4+1} \cdot (1-3) = -\frac{2}{5}$ **[A]**
- For $f(x)$ to be negative means the entire curve would be under the x-axis thus eliminating E. For $f'(x)$ to be negative would mean the curve is always decreasing thus eliminating A and C. For $f''(x)$ to be negative would mean the curve is always concave down eliminating D **[B]**
- $du = 2 dx$ or $dx = \frac{1}{2} du$ For limits of integration, if $x = 2$, then $u = 2(x) + 1 = 5$ and similarly when $x = 0$, $u = 1$. Doing all of the substitutions yields $\frac{1}{2} \int_1^5 \sqrt{u} du$ **[C]**

12. Rate of change of volume is $\frac{dV}{dt}$. When doing a direct proportion, say a is directly proportional to b, you get $a = kb$. Applying this gives $\frac{dV}{dt} = k\sqrt{V}$ **[E]**
13. Think of continuity as being able to draw the picture without lifting your pencil. This eliminates b, c; d. A derivative exists where you can draw a single tangent line to the curve, thus eliminating e. **[A]**
14. $x^2(2(\cos 2x)) + \sin 2x(2x)$ Factor out $2x$ giving $2x(x \cos 2x + \sin 2x) = 2x(\sin 2x + x \cos 2x)$ **[E]**
15. If a function is decreasing, then its derivative must be negative or less than zero. Notice that $x^2 - \frac{2}{x} < 0$ is trivially false for 0, and for $x < 0$, both x^2 and $-\frac{2}{x}$ are positive making $x^2 - \frac{2}{x} < 0$ false. So consider $x > 0$. Then $x^2 - \frac{2}{x} < 0$ is the same as $x^2 < \frac{2}{x}$ or $x^3 < 2$ or $x < \sqrt[3]{2}$. This makes $(0, \sqrt[3]{2}]$. There is controversy on whether the point where the derivative equals zero should be included in the interval. **[D]**
16. The derivative at the point is the slope of the tangent line at the point. Calculate the slope using the two-point formula $\frac{-2-7}{-2-1} = \frac{-9}{-3} = 3$ **[C]**
17. The graph would be concave down when the second derivative is negative. The first derivative is $2xe^x + 2e^x$. Using this as an aid, the second derivative is $2xe^x + 2e^x + 2e^x$ or $2xe^x + 4e^x$. Factor out e^x giving $e^x(2x + 4)$. As e^x is always positive, disregard it. Find when negative. $2x + 4 < 0 \rightarrow 2x < -4 \rightarrow x < -2$ **[A]**
18. Since it only has two zeros, the signs of $g'(x)$ in all intervals must not change. A function decreases when its derivative is negative or below the x-axis. From the table, this occurs between -2 and 2. **[A]**
19. The slope is the derivative, so the function would be the integral. The integral of this derivative is $y = x^2 + 3x + C$. Since the curve goes through (1,2) then $2 = 1^2 + 3(1) + C$ or $C = -2$. So $y = x^2 + 3x - 2$. **[D]**

20. For the limit to exist, it must have the same value as defined on both sides of 3. Since for $x + 2$ and $4x - 7$, when $x = 3$, they are 5, the limit exists. To be continuous, not only does the limit have to exist, but it must be defined to have the same value at 3 as the limit at 3. $f(3) = 5$ as defined. Thus the function is continuous. For it to be differentiable at 3 the derivatives as defined on both sides of 3 must have the same value. The derivatives are 1 and 4, so it is not differentiable. **[D]**
21. A function has an inflection point where the second derivative is 0 and its sign changes on opposite sides of this zero point (graphically, the curve has a point on the x-axis and the curve on one side of this zero point is above the x-axis and below the x-axis on the other side). There are 3 zero points. The one at b does not meet the criteria. **[A]**
22. If it is a line, then its equation must be $y = mx + b$ where m is the slope, or $f'(x)$. The slope is constant for a line. Using the two points (1,0) and (0,6) then the slope is $\frac{0-6}{1-0} = -6$. The y-intercept of this line is 6 from the graph. Thus the derivative is $-6x + 6$. The function will be the integral of this or $f(x) = -3x^2 + 6x + C$. Putting in $f(0) = 5$ gives $5 = -3(0)^2 + 6(0) + C$ or $C = 5$. Now $f(x) = -3x^2 + 6x + 5$ so $f(1) = 8$. **[D]**
23. This uses what is frequently listed as the Second Fundamental Theorem of Calculus. The derivative of an integral puts you back where you started when one of the limits is a variable expression and the other is a constant with the changes that the variable becomes the variable expression and you multiply everything by the derivative of the limit which is the variable expression (note: If the variable expression is not the upper limit, multiply your answer by a negative 1 to reverse the limit). So the answer will be $2x(\sin(x^2)^3)$ or $2x \sin(x^6)$ **[E]**
24. To get the equation of the tangent line you need a slope (derivative of the curve at the point) and a point. The derivative $f'(x) = 12x^2 - 5$ so the slope at $x = -1$ will be $f'(-1) = 7$. Using the original equation to find $f(-1)$ is 4 gives the point. The equation of the line is then $(y - 4) = 7(x - -1)$ or $y = 7x + 11$. **[C]**
25. A particle is at rest when its velocity (first derivative of the position) is 0. Taking the first derivative gives $6t^2 - 42t + 72$. Setting this to 0 and then factoring gives $6(t^2 - 7t + 12) = 6(t - 4)(t - 3) = 0$ so $v(t) = 0$ when $t = 3, 4$ **[E]**

26. Take the derivative implicitly. $6y \frac{dy}{dx} - 4x = -2x \frac{dy}{dx} - 2y$. Solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx}(6y + 2x) = 4x - 2y \rightarrow \frac{dy}{dx} = \frac{4x - 2y}{6y + 2x} \text{ Substituting in value for } x \text{ and } y \text{ will}$$

$$\text{give } \frac{dy}{dx} = \frac{4(3) - 2(2)}{6(2) + 2(3)} = \frac{12 - 4}{12 + 6} = \frac{8}{18} = \frac{4}{9} \quad \boxed{B}$$

27. $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ or $f'(x) = \frac{1}{f^{-1}'(y)}$ or $f'(x) = \frac{1}{g'(y)}$. Thus since $g(2) = 1$, then we

$$\text{can say that } f'(1) = \frac{1}{g'(2)} \text{ or } g'(2) = \frac{1}{f'(1)}. \quad f'(x) = 3x^2 + 1. \quad f'(1) = 4$$

$$\text{So } g'(2) = \frac{1}{4}. \quad \boxed{B}$$

28. The first derivative positive means the function is increasing and the second derivative positive means that the curve is concave up. A concave up increasing curve must have its slope line getting more vertical as x increases, so the rate of change must continue to grow as x increases. The rate of change of $g(x)$ from $x = 4$ to $x = 5$ was 6. The change then from $x = 5$ to $x = 6$ must be greater than 6. The only value showing this type of change would be $g(x) = 24$. E