D-CD3

1. Use the limit definition of derivative to show that for $f(x) = 2x^2 - 5x$, $f'(x) = 4x - 5$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h}$$

$$\lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 5x - 5h - 2x^2 + 5x}{h}$$

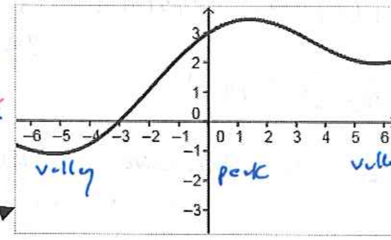
$$\lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{5x} - 5h \cancel{-2x^2} \cancel{+5x}}{h}$$

$$\lim_{h \to 0} \frac{4xh + 2h^2 - 5h}{h} \rightarrow \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 5)}{\cancel{h}}$$

$$\lim_{h \to 0} 4x + 2h - 5$$

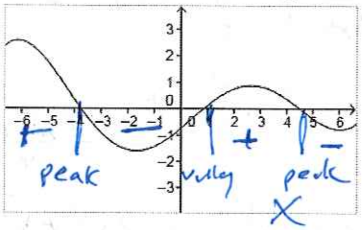
$$= 4x + 2(0) - 5$$

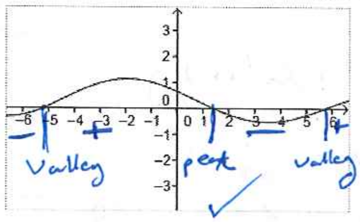
$$= 4x - 5$$

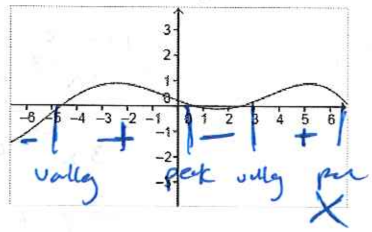


D-CD1

2. Which of the following could be the derivative of the given graph? Explain your choice.







(see my peak/valley marks)

D-CD4

3. Is the following function differentiable at x=0? Explain. $f(x) = \begin{cases} x^2 & , x \le 0 \\ -2x, & x > 0 \end{cases}$

• is $f(x)$ continuous? plug 0 into both.

$(0)^2 = 0 \qquad -2(0) = 0$ ✓ yes, f is cont.

• is $f'(x)$ continuous? $f'(x) = \begin{cases} 2x, & x \le 0 \\ -2, & x > 0 \end{cases}$

$\lim_{x \to 0^-} 2x = 0$

$\lim_{x \to 0^+} -2 = -2$

$\lim_{x \to 0} f'(x)$ d.n.e.

so $f'(x)$ is not continuous

F is not diff. @ $x=0$ b/c $f'(x)$ is not continuous at $x=0$.

D-CD5

4. Find the x-values where $f(x) = \frac{x^2}{x-3}$ has horizontal and vertical tangents.

Quot. rule $\quad f = x^2 \quad g = x-3 \quad \frac{f'g - fg'}{g^2} \Rightarrow \frac{(2x)(x-3) - x^2 \cdot 1}{(x-3)^2}$

$f' = 2x \quad g' = 1$

$$\frac{2x^2 - 6x - x^2}{(x-3)^2}$$

$$f'(x) = \frac{x^2 - 6x}{(x-3)^2}$$

Horiz. tangent

slope $= 0$ $\quad x^2 - 6x = 0$

$x(x-6) = 0$

$\boxed{x=0}$ $\boxed{x=6}$

Horiz. Tan

vertical tangents

slope is undefined; (div by 0.)

$(x-3)^2 = 0$

$x = 3$

vertical tangent

need

point slope

D-CD7

5. Write the equation of the line tangent to $f(x) = 3x^4\ \sqrt{5x-1}$ at x=2.

f g

• point

$f(2) = 3(2^4)\cdot\sqrt{5(2)-1} = 3\cdot16\cdot\sqrt{9}$

$3\cdot16\cdot3 = 144$

$f = 3x^4$ $\quad g = (5x-1)^{1/2}$

$f' = 12x^3$ $\quad g' = \frac{1}{2}\cdot(5x-1)^{-1/2}\cdot 5$

point $(2, 144)$

• slope take derivative: $f'(x) = 12x^3\cdot\sqrt{5x-1} + 3x^4\cdot\frac{1}{2}\cdot(5x-1)^{-1/2}\cdot 5$ ← No need to simplify.

Now plug in $x=2$ $\quad f'(2) = 12(2)^3\cdot\sqrt{9} + 3(2^4)\cdot\frac{1}{2}\cdot(9)^{-1/2}\cdot 5$

$12\cdot8\cdot3 + 3\cdot16\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot5 = \boxed{328}$ slope

$y - y_1 = m(x - x_1)$

$\boxed{y - 144 = 328(x-2)}$

D-AD2 and D-AD3

Find the following derivatives.

6. $y = -\frac{1}{x^3} + x^2 + \ln(2x)$ = rewrite $-x^{-3} + x^2 + \ln(2x)$ $\frac{d}{dx}$

$--3x^{-4} + 2x + \frac{1}{2x}\cdot 2$

$\frac{3}{x^4} + 2x + \frac{\cancel{2}}{\cancel{2}x} \Rightarrow \boxed{\frac{3}{x^4} + 2x + \frac{1}{x}}$

Can do more → $\frac{3 + 2x^5 + x^4}{x^4}$

$\frac{2x^5 + x^4 + 3}{x^4}$

7. $f(x) = e^{2x}\cos(4x)$ PRODUCT RULE

$f = e^{2x}$ $\quad g = \cos(4x)$

$f' = e^{2x}\cdot 2 = 2e^{2x}$ $\quad g' = -\sin(4x)\cdot 4 = -4\sin(4x)$

$f'g + fg'$

$2e^{2x}\cos(4x) + e^{2x}\cdot -4\sin 4x \Rightarrow 2e^{2x}\cos(4x) - 4e^{2x}\sin(4x)$ factor out...

$2e^{2x}(\cos(4x) - 2\sin(4x))$

8. $y = \cot^2(x^2)$ rewrite

$y = (\cot(x^2))^2$

$y' = 2(\cot(x^2))'\cdot -\csc^2(x^2)\cdot 2x$

$-4x\cot(x^2)\cdot\csc^2(x^2)$

$\boxed{y' = -4x\cot(x^2)\csc^2(x^2)}$

D-AD4

Use the table to find the derivatives

9. If $h(x) = [f(x)]^2$, find $h'(4)$

$h'(x) = 2[f(x)]^1\cdot f'(x)$

$h'(4) = 2[f(4)]\cdot f'(4)$ ⇓ table

$2[4]\cdot 1 = 8$

10. If $z(x) = f(g(x))$, find $z'(4)$

$z'(x) = f'(g(x))\cdot g'(x)$

$z'(4) = f'(g(4))\cdot g'(4)$

$f'(2)\cdot 1$

$f'(2)\cdot 1$

$1\cdot 1 = 1$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	1	4	-2
2	2	1	2	$-\frac{3}{2}$
3	3	1	1	0
4	4	1	2	1