

SOL5.

MATH 4000N

F-C1

Practice Assessment

1. Use the definition of continuity to show that $f(x)$ is continuous at $x=1$ $f(x) = \begin{cases} 3x+5 & x < 1 \\ 8x & x = 1 \\ 2x^2+6 & x > 1 \end{cases}$

$$\underset{x \rightarrow 1^-}{\lim} f(x) = f(1) = \underset{x \rightarrow 1^+}{\lim} f(x)$$

$$\underset{x \rightarrow 1^-}{\lim} 3x+5 = 8(1) = \underset{x \rightarrow 1^+}{\lim} 2x^2+6$$

$$3(1)+5 \qquad \qquad 2(1)^2+6$$

F-C2 $8 \leq 8 = 8 \Rightarrow$ continuous @ $x=1$.

2. Find the values of a and b that will make this function continuous everywhere:

$$f(x) = \begin{cases} x^2 - 5 & x < 0 \\ ax + b & 0 \leq x < 2 \\ 2x^2 - 6 & x \geq 2 \end{cases} \quad \text{"Find } f(x) \text{ at } x=0, x=2 \text{ and make it continuous"}$$

Continuous

@ $x=0$?

$$\underset{x \rightarrow 0^-}{\lim} f(x) = f(0) = \underset{x \rightarrow 0^+}{\lim} f(x)$$

$$\underset{x \rightarrow 0^-}{\lim} x^2 - 5 = a(0) + b = \underset{x \rightarrow 0^+}{\lim} ax + b$$

$$-5 = b = b \quad \text{So, } \underline{\underline{b = -5}}$$

$$\left\{ \begin{array}{l} \underset{x \rightarrow 2^-}{\lim} f(x) = f(2) = \underset{x \rightarrow 2^+}{\lim} f(x) \\ \underset{x \rightarrow 2^-}{\lim} ax + b = a(2) + b = \underset{x \rightarrow 2^+}{\lim} 2x^2 - 6 \\ a(2) + b = 2a + b = 2 \\ 2a - 5 = 2 \\ a = 3.5 \end{array} \right.$$

F-C3

3. Find and classify any discontinuities of the function. Justify your answer with limits.

$$f(x) = \frac{x-3}{2x^2 - 2x - 12} = \frac{x-3}{2(x^2 - x - 6)} = \frac{x-3}{2(x-3)(x+2)} = \frac{1}{2(x+2)}$$

~~$x-3$~~ REMOVABLE!

$x = 3$ is Removable

BECUSE

$$\lim_{x \rightarrow 3} \frac{1}{2(x+2)} = \frac{1}{2(3+2)} = \frac{1}{12}$$

(limit exists)

$x = -2$ is infinite because

$$\lim_{x \rightarrow -2^-} \frac{1}{2(x+2)} = \frac{1}{2(-2^-+2)} = \frac{1}{2(0^-)} = \frac{1}{0^-}$$

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