4.8 Definite and Indefinite Integrals

FIND THE FOLLOWING INDEFINITE INTEGRALS.

956. $\displaystyle\int (x^2-1)^2\,dx$

957. $\displaystyle\int \tfrac{1}{2}\cos 5x\,dx$

958. $\displaystyle\int 2^{5w}\,dw$

959. $\displaystyle\int \sin(5\theta)\cos(5\theta)\,d\theta$

960. $\displaystyle\int \frac{4x}{(4x^2-1)^5}\,dx$

961. $\displaystyle\int \frac{z^3-2z^2-5}{z^2}\,dz$

962. $\displaystyle\int (x^2+14x+49)^{35}\,dx$

963. $\displaystyle\int e^x(e^x-1)^7\,dx$

964. $\displaystyle\int [\sin(5\theta)+1]^4\cos(5\theta)\,d\theta$

965. $\displaystyle\int 2^{\log_2 7x}\,dx$

FIND EXACT VALUES FOR THE FOLLOWING DEFINITE INTEGRALS.

966. $\displaystyle\int_{-1}^{1} x(x^2+1)^3\,dx$ $= 0$

967. $\displaystyle\int_{0}^{1} x\sqrt{1-x^2}\,dx$ $= 1/3$

968. $\displaystyle\int_{0}^{4} \frac{1}{\sqrt{2x+1}}\,dx$ $= 2$

969. $\displaystyle\int_{0}^{2} \frac{x}{\sqrt{1+2x^2}}\,dx$ $= 1$

970. $\displaystyle\int_{1}^{9} \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$ $= 1/2$

971. $\displaystyle\int_{0}^{2} x\sqrt[3]{x^2+4}\,dx$ $\frac{3}{8}(16-\sqrt[3]{256})$

972. $\displaystyle\int_{1}^{2} (x-1)\sqrt{2-x}\,dx$ $= 4/15$

973. $\displaystyle\int_{0}^{4} \frac{x}{\sqrt{2x+1}}\,dx$

974. $\displaystyle\int_{0}^{\pi/2} \cos(\tfrac{2x}{3})\,dx$

975. $\displaystyle\int_{\pi/3}^{\pi/2} (x+\cos x)\,dx$

976. $\displaystyle\int_{0}^{7} x\sqrt[3]{x+1}\,dx$

977. $\displaystyle\int_{-2}^{6} x^2\sqrt[3]{x+2}\,dx$

FIND THE AREA UNDER THE CURVE OVER THE GIVEN INTERVAL.

978. $y = 2\sin x + \sin(2x);\quad [0,\pi]$

979. $y = \sin x + \cos(2x);\quad [0,\pi]$

980. $y = \sec^2(\tfrac{x}{2});\quad [\tfrac{\pi}{2},\tfrac{2\pi}{3}]$

981. $y = \csc(2x)\cot(2x);\quad [\tfrac{\pi}{12},\tfrac{\pi}{4}]$

No one really understood music unless he was a scientist, her father had declared, and not just any scientist, either, oh, no, only the real ones, the theoreticians, whose language is mathematics. She had not understood mathematics until he had explained to her that it was the symbolic language of relationships. "And relationships," he had told her, "contained the essential meaning of life." —*Pearl S. Buck,* The Goddess Abides, Part 1

4.9 Integrals Involving Logarithms and Exponentials

Find the following indefinite integrals.

982. $\displaystyle\int \frac{1}{x+1}\,dx$

983. $\displaystyle\int \frac{x}{x^2+1}\,dx$

984. $\displaystyle\int \frac{x^2-4}{x}\,dx$

985. $\displaystyle\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$

986. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

987. $\displaystyle\int \frac{1}{\sqrt{x+1}}\,dx$

988. $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}-3}\,dx$

989. $\displaystyle\int \frac{2x}{(x-1)^2}\,dx$

990. $\displaystyle\int \frac{\cos\theta}{\sin\theta}\,d\theta$

991. $\displaystyle\int \csc(2\theta)\,d\theta$

992. $\displaystyle\int \frac{\cos\theta}{1+\sin\theta}\,d\theta$

993. $\displaystyle\int \frac{\sec\theta\tan\theta}{\sec\theta-1}\,d\theta$

994. $\displaystyle\int 5e^{5x}\,dx$

995. $\displaystyle\int \frac{e^{-x}}{1+e^{-x}}\,dx$

996. $\displaystyle\int e^x\sqrt{1-e^x}\,dx$

997. $\displaystyle\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx$

998. $\displaystyle\int \frac{5-e^x}{e^{2x}}\,dx$

999. $\displaystyle\int e^{\sin(\pi x)}\cos(\pi x)\,dx$

1000. $\displaystyle\int e^{-x}\tan(e^{-x})\,dx$

1001. $\displaystyle\int 3^x\,dx$

1002. $\displaystyle\int 5^{-x^2}x\,dx$

1003. $\displaystyle\int \frac{3^{2x}}{1+3^{2x}}\,dx$

Find exact values for each of the following definite integrals.

1004. $\displaystyle\int_0^4 \frac{5}{3x+1}\,dx$ $= \frac{5}{3}\ln 13$

1005. $\displaystyle\int_{-1}^1 \frac{1}{x+2}\,dx$ $= \ln 3$

1006. $\displaystyle\int_e^{e^2} \frac{1}{x\ln x}\,dx$ $= \ln 2$

~~**1007.**~~ $\displaystyle\int_0^2 \frac{x^2-2}{x+1}\,dx$ Requires partial fract. decomposition (Sorry.)

1008. $\displaystyle\int_\pi^{2\pi} \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$

1009. $\displaystyle\int_1^5 \frac{x+5}{x}\,dx$

1010. $\displaystyle\int_0^1 e^{-2x}\,dx$

1011. $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\,dx$

1012. $\displaystyle\int_{-1}^2 2^x\,dx$

1013. $\displaystyle\int_0^1 \frac{3^{4x}(4\ln 3)}{3^{4x}+1}\,dx$

966. $\boxed{\frac{1}{2}\int_{-1}^{1} 2}\, x(x^2+1)^3\,dx$

$\frac{1}{2}\left[\frac{1}{4}(x^2+1)^4\right]_{-1}^{1} \rightarrow \frac{1}{2}\left(\frac{1}{4}(\cancel{2})^{\cancel{4}\,16} - \frac{1}{4}(\cancel{2})^{\cancel{4}\,16}\right) = \frac{1}{2}(0) = 0$

967. $\boxed{-\frac{1}{2}\int_0^1 -2}\, x\sqrt{1-x^2}\,dx$

$-\frac{1}{2}\left[\frac{2}{3}(1-x^2)^{3/2}\right]_0^1 \rightarrow -\frac{1}{2}\left[\cancel{\frac{2}{3}(0)^{3/2}} - \frac{2}{3}(1)^{3/2}\right] = -\frac{1}{2}\left(-\frac{2}{3}\right) = 1/3$

968.) $\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int_0^4 2\,(2x+1)^{-1/2}\,dx$

$\frac{1}{2}\left[2(2x+1)^{1/2}\right]_0^4 = \frac{1}{2}\left[2(\cancel{9})^{\cancel{1/2}\,3} - 2(1)^{1/2}\right]$

$\frac{1}{2}\left[6-2\right] = \frac{1}{2}(4)$

$= 2$

969.) $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = \frac{1}{4}\int_0^2 4x\,(1+2x^2)^{-1/2}$

$\frac{1}{4}\left[2(1+2x^2)^{1/2}\right]_0^2 \rightarrow \frac{1}{4}\left[2(9)^{1/2} - 2(1)^{1/2}\right]$

$\frac{1}{4}\left[6-2\right] = \frac{1}{4}(4) = 1$

970) $\int_1^9 \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx = 2\int_1^9 \frac{1}{2}x^{-1/2}(1+x^{1/2})^{-2}$

$2\left[-1(1+x^{1/2})^{-1}\right]_1^9 = 2\left[\frac{-1}{1+x^{1/2}}\right]_1^9 = 2\left[\left(\frac{-1}{1+3}\right)-\left(\frac{-1}{1+1}\right)\right]$

$2\left[-\frac{1}{4}+\frac{1}{2}\right] = 2\left(\frac{1}{4}\right) = 1/2$

971) $\int_0^2 x\sqrt[3]{x^2+4}\,dx = \frac{1}{2}\int_0^2 2x(x^2+4)^{1/3}$

$\frac{1}{2}\left[\frac{3}{4}(x^2+4)^{4/3}\right]_0^2 = \frac{1}{2}\left(\frac{3}{4}(8)^{4/3} - \frac{3}{4}(4)^{4/3}\right)$

$(\sqrt[3]{8})^4 \qquad \sqrt[3]{4^4}$

$(2)^4 \qquad \sqrt[3]{256}$

$\frac{1}{2}\left(\frac{3}{4}(16) - \frac{3}{4}(\sqrt[3]{256})\right)$

$\frac{1}{2}\cdot\frac{3}{4}(16-\sqrt[3]{256})$

$\frac{3}{8}(16-\sqrt[3]{256})$

972. $\int_1^2 (x-1)\sqrt{2-x}\,dx$

No Rev. Chain Rule... <u>u-sub</u>.

Limits of Integr.

$\int_1^2$ these are x's. Change them to u's.

$u=2-x$

$\int_1^0$

Let $u=2-x$

$\Rightarrow \frac{du}{dx}=-1 \Rightarrow dx=-du$

$\Rightarrow x=2-u$

$\int_1^0 (2-u-1)(u)^{1/2}\,du$

$\int_1^0 (1-u)u^{1/2}\,du$

$\int_1^0 u^{1/2}-u^{3/2}\,du$

$\left[\frac{2}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]_1^0 = 0-\left(\frac{2}{3}(1)-\frac{2}{5}(1)\right)$

$0-\left(\frac{2}{3}-\frac{2}{5}\right)$

$\frac{10}{15}-\frac{6}{15}$

$-\left(4/15\right) = -4/15 = 4/15$

$$1004.\ \int_0^4 \frac{5}{3x+1}\,dx = \frac{5}{3}\int_0^4 \frac{3}{5}\,5\cdot\frac{1}{3x+1}\,dx$$

3

$$\frac{5}{3}\Big[\ln|3x+1|\Big]_0^4 = \frac{5}{3}\left(\ln 13 - \cancel{\ln 1}^{\,0}\right)$$

$$\frac{5}{3}\ln 13 = \ln\sqrt[3]{13^5}$$

1005:

$$\int_{-1}^{1}\frac{1}{x+2}\,dx$$

$$= \ln|x+2|\Big]_{-1}^{1} = \ln 3 - \cancel{\ln 1}^{\,0} = \boxed{\ln 3}$$

$$1006)\ \int_e^{e^2}\frac{1}{x\ln x}\,dx = \int_e^{e^2}\frac{1}{x}\cdot\frac{1}{\ln x}\,dx = \int_e^{e^2}\frac{1}{x}(\ln x)^{-1}\,dx$$

$$\ln|\ln|x||\Big]_e^{e^2} = \ln|\ln e^2| - \ln|\ln e|$$

$$\ln 2 - \cancel{\ln 1}^{\,0}$$

$$\ln 2$$