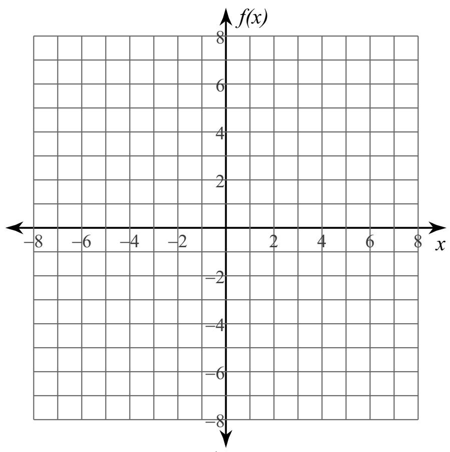
AP Calculus Name____________________________________

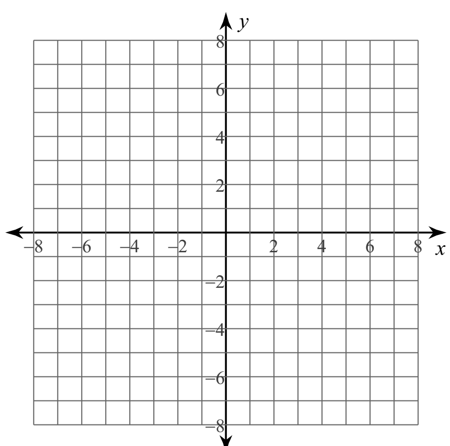
Winter Break Curve Sketching Packet of Misery Date________________

Find x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative extrema. Then graph. No calculators.

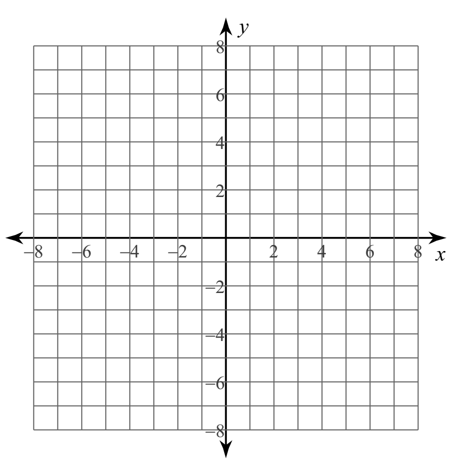
1) $f(x) = (5x - 15)^{\frac{2}{3}}$



2) $y = (6x - 12)^{\frac{1}{3}}$



3) $y = \frac{x^3}{3} - \frac{x^2}{3} - \frac{8x}{3}$



Curve Sketching

GUIDELINES FOR SKETCHING A CURVE:

A. Domain.

B. Intercepts: x- and y-intercepts.

C. Symmetry: even ($f(-x) = f(x)$) or odd ($f(-x) = -f(x)$) function or neither, periodic function.

D. Asymptotes: horizontal $\left(\lim\limits_{x\to\pm\infty} f(x) = L\right)$ and vertical $\left(\lim\limits_{x\to a^{\pm}} f(x) = \pm\infty\right)$ asymptotes.

E. Intervals of Increase or Decrease: Number line method with the critical no's and F'

F. Local Maximum and Minimum Values: Use critical numbers and number line method.

G. Concavity and Points of Inflection: Compute $f''(x)$ and use the Concavity Test.

H. Sketch the Curve: Using the information in items A-G, draw the graph.

EXAMPLE: Use the guidelines to sketch the curve $f(x) = \dfrac{x-1}{x+2}$.

Solution:

A. Domain: $\boxed{(-\infty,-2)\cup(-2,\infty)}$

B. Intercepts:

(i) x-intercept ($y = 0$):

$$\frac{x-1}{x+2} = 0 \quad\Longrightarrow\quad x-1=0 \quad\Longrightarrow\quad x=1 \quad\Longrightarrow\quad \boxed{(1,0)}$$

(ii) y-intercept ($x = 0$):

$$y = \frac{0-1}{0+2} = -\frac{1}{2} \quad\Longrightarrow\quad \boxed{\left(0,-\frac{1}{2}\right)}$$

C. Symmetry:

(i) This function is $\boxed{\text{not even}}$, since

$$f(-1) = \frac{-1-1}{-1+2} \neq \frac{1-1}{1+2} = f(1)$$

(ii) This function is $\boxed{\text{not odd}}$, since

$$f(-1) = \frac{-1-1}{-1+2} \neq -\frac{1-1}{1+2} = -f(1)$$

(iii) This function is $\boxed{\text{not periodic}}$.

D. Asymptotes:

Note from Mohyuddin: Can also just use degree/coefficient shortcut.

(i) Horizontal asymptote:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x-1}{x+2} = \lim_{x\to\pm\infty} \frac{\frac{x-1}{x}}{\frac{x+2}{x}}$$

$$= \lim_{x\to\pm\infty} \frac{1-\frac{1}{x}}{1+\frac{2}{x}} = 1 \quad \Longrightarrow \quad \boxed{y = 1 \text{ is the horizontal asymptote}}$$

(ii) Vertical asymptote:

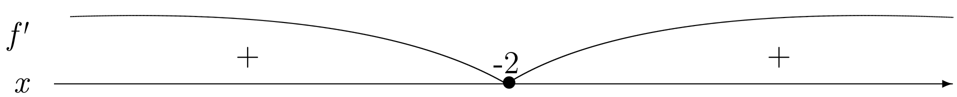
$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{x-1}{x+2} = \left[\begin{array}{l} \text{WORK:} \\ \dfrac{-2.01-1}{-2.01+2} = \dfrac{-3.01}{-0.01} = \dfrac{-\text{“NOT SMALL”}}{-\text{“SMALL”}} = +\text{“BIG”} \end{array} \right] = \infty$$

Therefore $\boxed{x = -2 \text{ is the vertical asymptote}}$.

E. Intervals of Increase or Decrease: We have

$$f'(x) = \left(\frac{x-1}{x+2}\right)' = \frac{(x-1)'(x+2) - (x-1)(x+2)'}{(x+2)^2} = \frac{1\cdot(x+2) - (x-1)\cdot 1}{(x+2)^2}$$

$$= \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2} > 0$$

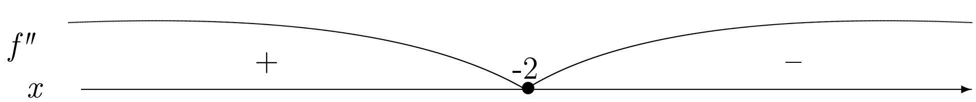


It follows that this function $\boxed{\text{increases on } (-\infty,-2) \text{ and } (-2,\infty)}$.

F. Local Maximum and Minimum Values: This function has no critical numbers, therefore it has $\boxed{\text{no local maximum and minimum values}}$.

G. Concavity and Points of Inflection: We have

$$f''(x) = \left(\frac{3}{(x+2)^2}\right)' = \left(3(x+2)^{-2}\right)' = 3\left((x+2)^{-2}\right)' = -6(x+2)^{-3} = -\frac{6}{(x+2)^3}$$

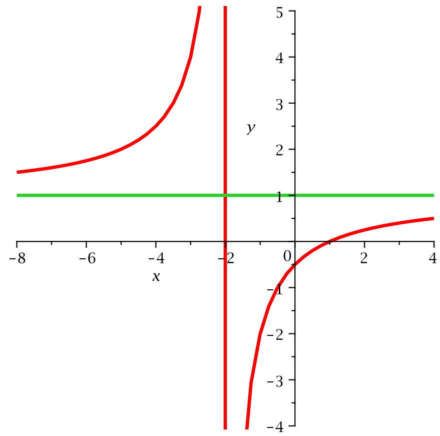


It follows that $f''(x) > 0$ if $x < -2$ and $f''(x) < 0$ if $x > -2$, therefore $f(x)$ is

$$\boxed{\text{concave upward on } (-\infty,-2)} \text{ and } \boxed{\text{concave downward on } (-2,\infty)}$$

There are $\boxed{\text{no inflection points}}$, since $x = -2$ is not in the domain.

H. Sketch the Curve: We have



EXAMPLE: Use the guidelines to sketch the curve $f(x) = xe^{-x^2/2}$

Solution:

A. Domain:

B. Intercepts:

(i) x-intercepts:

(ii) y-intercepts:

C. Symmetry:

(i) This function is even (?)

(ii) This function is odd (?)

(iii) This function is periodic (?)

D. Asymptotes:

(i) Horizontal asymptotes:

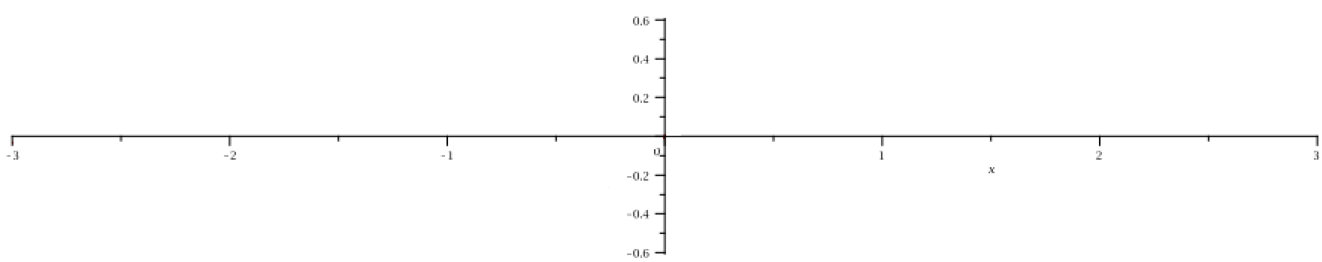
(ii) Vertical asymptotes:

E. Intervals of Increase or Decrease:

F. Local Maximum and Minimum Values:

G. Concavity and Points of Inflection:

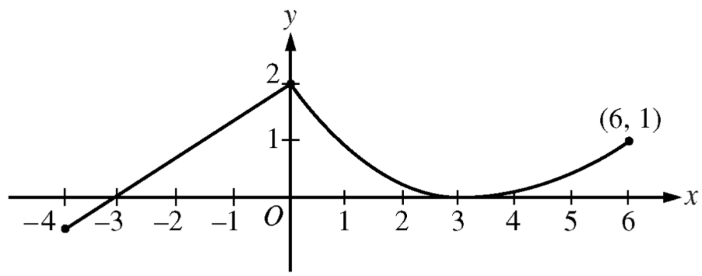
H. Sketch the Curve:



2008 AB 6 (Form B)

Consider the closed curve in the xy-plane given by $x^2 + 2x + y^4 + 4y = 5$.

a. Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.
b. Write an equation for the line tangent to the curve at the point (-2,1).
c. Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.
d. Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.