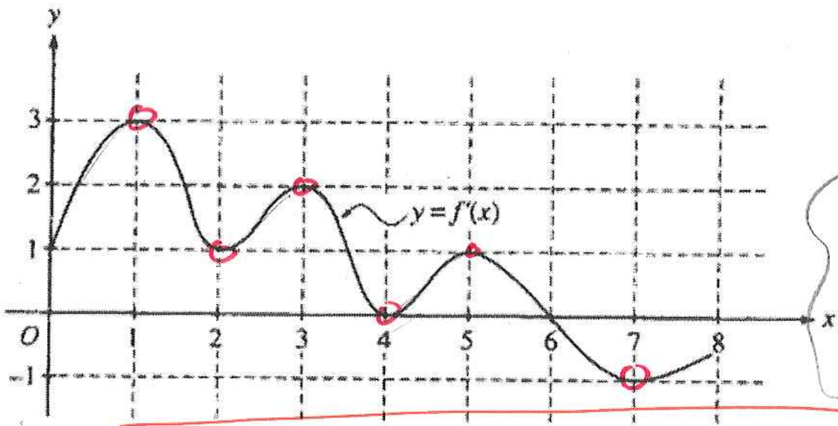
D-AD10

Last practice assessment of 2018!



1. Shown here is the graph of the first derivative of some function $f(x)$. Is f concave up or concave down over the interval (6,7)? Justify your response.

Concave up: f'' positive
concave down: f'' negative
f'' from f' graph? look at f' slope!
→ f' decreasing → f'' negative
f' increasing → f'' positive

#1 f is concave down on $(6,7)$ because f'' is negative [or f' is decreasing].

2. Using the above graph, give an interval for which f is increasing and concave up.

f' positive f' increasing, or, f'' positive

Any of:
$(0,1)$ $(2,3)$ $(4,5)$
#2

3. How many inflection points does f have over the interval [1,8]? Explain.

#3
6. There are 6 pts where f' changes from inc to dec. or vice versa

sign change in f'' → f' change inc ⇄ dec. → rel. extrema of f'

For 4 and 5, refer to the function $f(x) = 2x^5 - \frac{10}{3}x^4 + x$. Calculations/analysis do not need to be repeated if you need to make reference to the other problem.

D-AD11

4. Find any inflection points for the function $f(x)$. Justify your response.

$f'' = 0$ (or undef) and sign change

$f'(x) = 10x^4 - \frac{40}{3}x^3 + 1$
$f''(x) = 40x^3 - 40x^2 = 0$
$40x^2(x-1) = 0$
$x = 0 \quad x = 1$
Terrace pts

f: C.D. C.D C.U.
f'': − − +
$-1 \ldots \quad 0 \quad 1/2 \quad 1 \quad 1 \ldots$

$f''(1/2) = -$
$f''(-1\ldots) = -$
$f''(1\ldots) = +$

#4
f has an inflection point when $x = 1$ b/c f'' changes sign there.

5. Find the interval(s) over which $f(x)$ is concave up. Justify your response.

See previous sign chart.

f is concave up over $(1, \infty)$ b/c f'' is positive there.
#5

D-AD12

f' neg. $\quad f''$ neg.

6. Find any intervals for which $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 - 2$ is decreasing and concave down. Show the calculations that lead to your conclusion.

(CN) $f'(x) = \frac{1}{2}x^2 + x = 0$

$x(\frac{1}{2}x + 1) = 0$

$x = 0 \quad x = -2$

C.N.

(TP) $f''(x) = x + 1 = 0$

$x = -1$

T.P.

$f''(-100) = -$

$f''(1) = +$

f': $+ \quad - \quad +$; f'': $- \quad + $; $-2 \quad -1 \quad 0$

$f'(-1) = -$ $\quad f'(-100) = +$

$f'(1) = +$

$\boxed{f \text{ is dec. and C.D. on } (-2, -1)}$

D-CD8

7. State why the MVT is applicable to $f(x) = \sqrt{x}$ on the interval [0,4] Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x)$.

$f'(x) = \frac{1}{2}x^{-1/2} \rightarrow \frac{1}{2\sqrt{x}}$

MVT applies b/c f is differentiable on $(0, 4)$.

$\frac{f(b) - f(a)}{b - a} = f'(c)$ $\quad \frac{f(4) - f(0)}{4 - 0} = f'(c)$

$\frac{2 - 0}{4} = \frac{1}{2\sqrt{c}} \rightarrow \frac{1}{2} \times \frac{1}{2\sqrt{c}} \rightarrow 2\sqrt{c} = 2$

$\sqrt{c} = 1$

$c = \pm 1 \rightarrow c = 1$

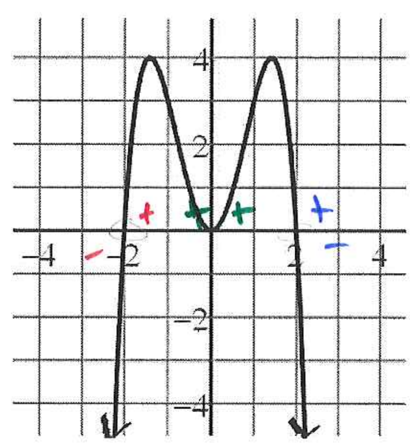
D-AD7: Use the graph for #7 and 8.

8. Shown here is $f'(x)$ the first derivative of $f(x)$. Give any interval(s) where $f(x)$ is decreasing. Justify.

f dec $\rightleftarrows$ f' negative

f inc $\rightleftarrows$ f' positive

#8 f is decreasing on $(-\infty, -2)$ and $(2, \infty)$ b/c f' is negative on these regions.



f' is graphed above

9. Does f have 2 or 3 relative extrema? Explain in detail, including locations and classifications.

f has 2 rel. extrema, but 3 critical numbers. These are at $x = -2, 0, 2$.

$x = -2$ is a rel min b/c f' changes from neg. → pos.

Conversely, f has a rel. max @ $x = 2$ b/c f' changes from pos → neg.

While f' is equal to zero @ $x = 0$, there is no sign changes so neither a relative max nor min there.

D-AD8

10. Find the absolute extrema for $f(x) = x^4 - 2x^2 - 3$ on the interval [-2,0]

Plug endpoints and C.N. into f

C.N. $f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$4x(x+1)(x-1) = 0$

$\underline{x=0}$ C.N. $\quad \cancel{x=1}$ not in interval $\quad \underline{x=-1}$ C.N.

$f(0) = -3$

$f(-2) = (-2)^4 - 2(-2)^2 - 3$

$16 - 2 \cdot 4 - 3$

$16 - 8 - 3$

$8 - 3 \rightarrow 5$ MAX

$f(-1) = (-1)^4 - 2(-1)^2 - 3$

$1 - 2 - 3$

$-1 - 3 \rightarrow -4$ MIN

ABS MAX $(-2, 5)$ $\qquad$ ABS MIN $(-1, -4)$

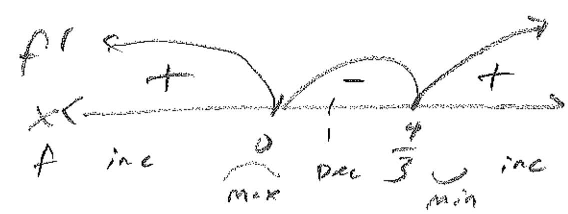
11. Find and classify any relative extrema. Justify your classifications. $f(x) = x^3 - 2x^2 + 4$

C.N. $f'(x) = 3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0 \qquad x = \frac{4}{3}$

C.N.



$f'(-1{,}000) = +$

$f'(1) = -$

$f'(1{,}000) = +$

f has a rel MAX when $x = 0$ b/c f' changes sign from positive to negative.

f has a rel. MIN when $x = \frac{4}{3}$ b/c f' changes sign from negative to positive.

D-AD9

12. For what interval(s) is the function $f(x) = -x^3 + 3x^2 - 3$ increasing? Justify your answer.

f' positive

CN.

$f'(x) = -3x^2 + 6x = 0$

$-3x(x-2) = 0$

$x = 0 \quad x = 2$

CN

f': $-$ $+$ $-$

x: 0 2

f: DEC INC DEC

$f'(1) = +$

$f'(100) = -$

$f'(-100) = -$

$\boxed{f \text{ is Increasing over } (0,2) \text{ b/c } f' \text{ is positive there.}}$

D-AD5

13. Find the slope of the line tangent to $3 - 2x^2 = 4x^3 + 5x^3y + 2x^2$ at the point $(-1, -1)$.

$\underbrace{5x^3}_{f}\,\underbrace{y}_{g}$ product rule

$f: 5x^3 \quad g: y$

$f': 15x^2 \quad g': y'$

$0 - 4x = 12x^2 + 15x^2y + 5x^3y' + 4x$

$-8x - 12x^2 - 15x^2y = 5x^3y'$

$\dfrac{-8x - 12x^2 - 15x^2y}{5x^3} = y'$

plug in $(-1,-1)$

$\dfrac{-8(-1) - 12(-1)^2 - 15(-1)^2(-1)}{5(-1)^3}$

$\dfrac{8 - 12 + 15}{-5} \longrightarrow \dfrac{-4+15}{-5} = -\dfrac{11}{5}$

Author's Note: I just realized $(-1,-1)$ does not lie on this curve. ¯_(ツ)_/¯