

D-AD15

Show all work for full credit

Practice Assessment

1.

The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V ? ($S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

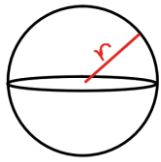
(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

Diagram:

Rates:

Equation:

Differentiate



$$\frac{dr}{dt} = 0.3$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi * 3r^2 \frac{dr}{dt}$$

$$S = 100\pi$$

$$\frac{dV}{dt} = ?$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substitute and Solve:

Need dV/dt ...have dr/dtdon't have r though. BUT! $4\pi r^2$ as a whole represents surface area! So given that $S = 100\pi = 4\pi r^2$ just replace 100π in for $4\pi r^2$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

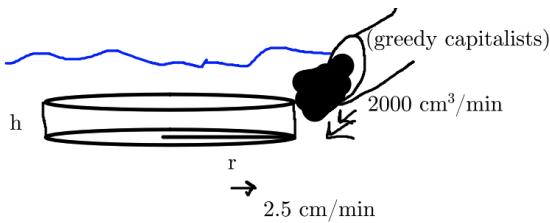
$$\frac{dV}{dt} = 100\pi * 0.3 \rightarrow 30\pi \quad E$$

2.

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

Diagram



Rates

$$\frac{dV}{dt} = 2000 \quad \frac{dr}{dt} = 2.5 \quad \frac{dh}{dt} = ?$$

$$r = 100$$

$$h = 0.5$$

Equation

$$V = \pi r^2 h$$

Differentiate

Use the product rule!!! R and H are variables!

$$\frac{dV}{dt} = \pi \left(2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right)$$

Commented [MN1]: SHOULD NOT BE HERE SORRY

Substitute and Solve

All is either known or needed ☺

$$\frac{dV}{dt} = \pi \left(2r \frac{dr}{dt} h + r^2 \frac{dh}{dt} \right)$$

Commented [MN2]: AHHHHH GO AWAY

$$2000 = \pi \left(2 * 100 * 2.5 * 0.5 + 100^2 * 2 * 0.5 * \frac{dh}{dt} \right)$$

Commented [MN3]: THANKFULLY 2 times 1/2 is 1 and it kind of goes away!!!!

$$\frac{2000}{\pi} = 250 + 10,000 \frac{dh}{dt}$$

$$636.620 = 250 + 10000 \frac{dh}{dt}$$

$$386.620 = 10000 \frac{dh}{dt} \rightarrow 0.039 = \frac{dh}{dt}$$

Commented [MN4]: WRONG DECIMAL PLACE SORRY