D-CD8 Practice Assessment

differentiable, so M.V.T. Applies

1. If c is the value that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval [0,2], then what is the value of c?

Avg = Instant.

Avg: $f(0) = 0$ $(0,0)$

$f(2) = 2^3 - 2\cdot 2^2 = 0$ $(2,0)$

slope: $\frac{0-0}{2-0} = \frac{0}{2} = 0$ ← avg. rate

Inst: $f'(x) = 3x^2 - 4x$

M.V.T.: Avg = Instant

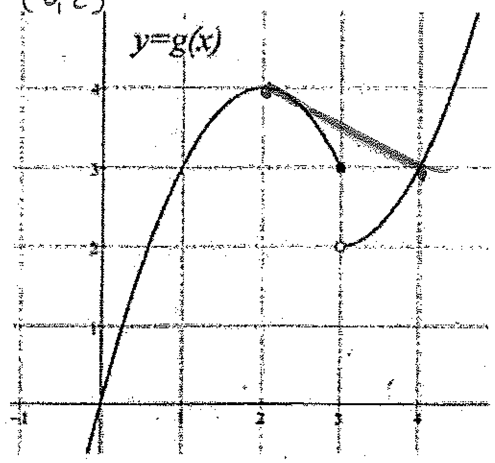
$0 = 3x^2 - 4x$

$0 = x(3x-4)$

$x = 0$ Not in (0,2) $x = \frac{4}{3}$ ✓

$\boxed{c = \frac{4}{3}}$

2. Explain why the function g(x) does not have a tangent line parallel to the secant line over [2,4]. Include a sketch of the secant line.



The M.V.T. says if a function is differentiable on an interval, then its average rate = instantaneous rate at some point in the interval, called c. But this function is not continuous, so also not differentiable, so M.V.T. doesn't apply.

Curve Sketching: Consider the function $f(x) = \frac{x-1}{x^2}$.

Use of a calculator only for CHECKING answer; all calculations/analysis must be shown.

Required questions:

3. What is the domain?

All $x \in \mathbb{R}$, $x \neq 0$

4. What are the coordinates of the x-intercept (if any)? Of the y-intercept?

$y = 0$

$0 = \frac{x-1}{x^2} \rightarrow x = 1$ $(1,0)$

$x = 0$ but can't... So, None.

5. Find the location of any vertical asymptotes by using limits. $\boxed{x = 0}$

$\lim_{x \to 0^-} \frac{x-1}{x^2} = \frac{0^- - 1}{(0^-)^2} = \frac{-}{0^+} = -\infty$; $\lim_{x \to 0^+} \frac{x-1}{x^2} = \frac{0^+ - 1}{(0^+)^2} = \frac{-}{0^+} = -\infty$

6. Find the location of any horizontal asymptotes by using limits.

$\boxed{y = 0}$

$\lim_{x \to \infty} \frac{x-1}{x^2} \Rightarrow \frac{\infty - 1}{\infty^2} = 0$

degree on bottom bigger → it grows faster

$f' = 1$

$g' = 2x$

$$f(x) = \frac{x-1}{x^2} \qquad f'(x) = \frac{1 \cdot x^2 - (x-1)\cdot 2x}{(x^2)^2} = \frac{x^2 - (2x^2 - 2x)}{x^4} = \frac{x^2 - 2x^2 + 2x}{x^4} = \frac{-x^2 + 2x}{x^4}$$

D-AD9 D-AD8

7. Find intervals of increase and decrease and classify local extrema. Find the (x,y) coordinates of any extrema.

$$f'(x) = -\frac{x-2}{x^3} = 0 \qquad -\frac{\cancel{x}(x-2)}{x^{\cancel{4}3}}$$

f' can define

$x - 2 = 0$

$x = 0$ C.N. $x = 2$ C.N.

$f' = 0$

f': − + −

x: 0 2

f: DEC 0 INC 2 DEC

~~MIN~~ MAX

No point @ x = 0

f'

$$\boxed{-\frac{x-2}{x^3}}$$

$f(2) = \frac{2-1}{2^2} = \frac{1}{4}$ $(2, 1/4)$

D-AD12 D-AD11

8. Find the intervals where the function is concave up, concave down, and the (x,y) coordinates of any inflection points.

$$f'' = -\frac{1 \cdot x^3 - (x-2)\cdot 3x^2}{x^6} = -\frac{x^3 - 3x^3 + 6x^2}{x^6} = -\frac{-2x^3 + 6x^2}{x^6}$$

$$f'' = -\frac{-2(x-3)}{x^4} \leftarrow -\frac{-2\cancel{x^2}(x-3)}{x^{\cancel{6}4}}$$

$$= \frac{2(x-3)}{x^4}$$

Terrace pts: $x = 3$, $x = 0$

$\boxed{\#7}$
- f increasing over $(0, 2)$ b/c f' positive
- f decreasing $(-\infty, 0)$ and $(2, \infty)$ b/c f' negative
- f has rel. max @ $(2, 1/4)$ b/c f' changes + → −

D-AD13

9. Over what intervals is:
   a. *f* increasing concave up?
   b. *f* decreasing concave up?
   c. *f* increasing concave down?
   d. *f* decreasing concave down?

f'': − − +

x: 0 3

f: C.D. 0 C.D. 3 C.U.

I.P.

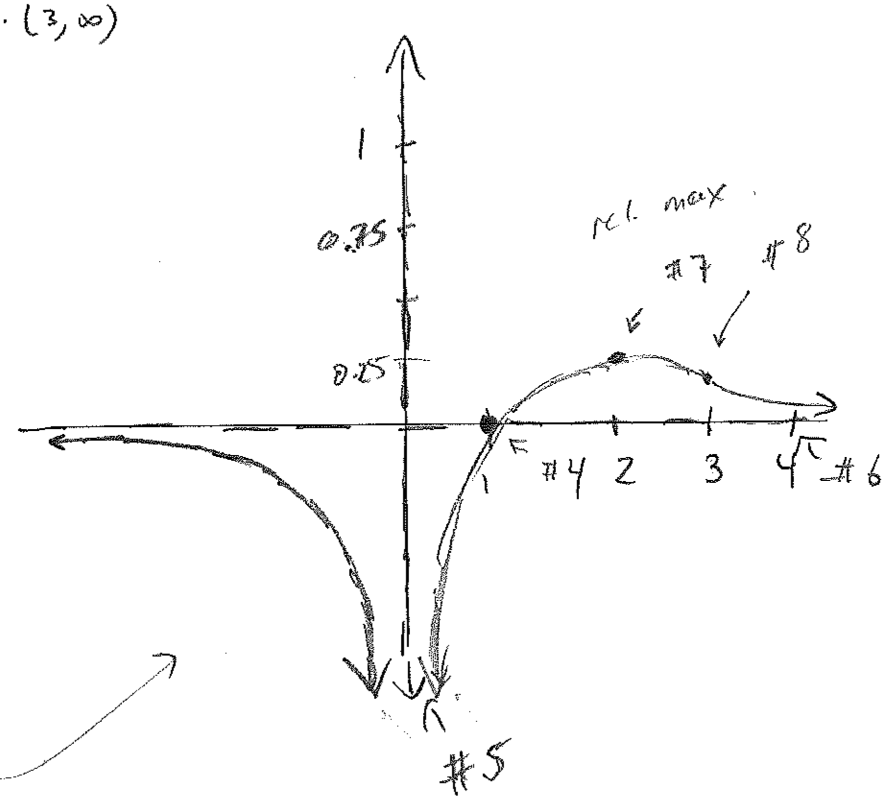
C.D. $(-\infty, 0)$

C.D. $(0, 3)$

C.U. $(3, \infty)$

$f(3) = \frac{3-1}{3^2} = \frac{2}{9}$ $\boxed{(3, 2/9) \text{ I.P.}}$

10. Graph the function using the points/analysis you found:



f'': − − − +

f': − + − −

f: DEC 0 INC 2 DEC 3 DEC

C.D. C.D. ↑ C.D. ↑ C.U.

↓ ↓ C.N. ↓ T.P. ↓