

Good afternoon: no warm up, we'll randomize and go over hw,  
then learn more about maxima and minima

Assessment: Monday

HW solutions at [mcalc.weebly.com](http://mcalc.weebly.com)

any in particular you had trouble with??

$$596.) \quad \frac{3x}{f} \cdot \frac{\csc 2x}{g}$$

$$3 \csc 2x + 3x \cdot -\csc 2x \cot 2x \cdot 2$$

$$3 \csc 2x - 6x \csc 2x \cot 2x$$

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Find the x-coordinate(s) where  $y = x^4 - 4x^2 + 1$  has relative extrema. Justify your classifications.

$$y' = 4x^3 - 8x = 0$$

$$4x(x^2 - 2) = 0$$

$$x = 0$$

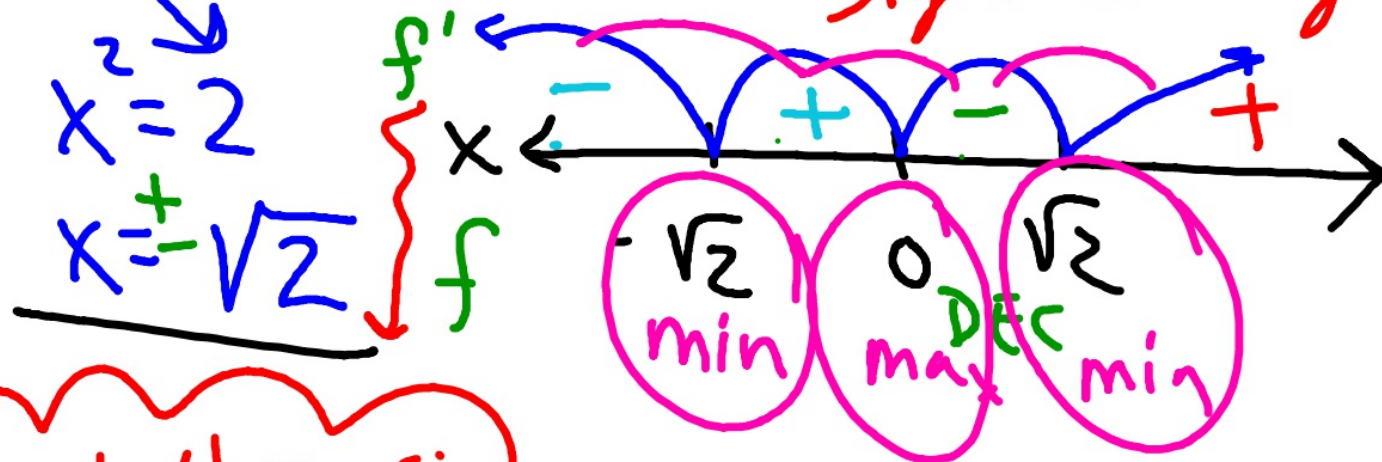
$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

C.N.

① Find C.N.

② Look for sign changes



Where do those signs come from?

Pick numbers in each interval. Plug into  $f'$ . Find sign of result.

$$\begin{aligned} f'(1) &= -4 \\ f'(-1) &= -4 \\ f'(1000) &= + \\ f'(-1000) &= - \end{aligned}$$

## How to find where a function has relative extrema

1. Take the derivative of  $y$ ,  $y'$

2. Find C.N.

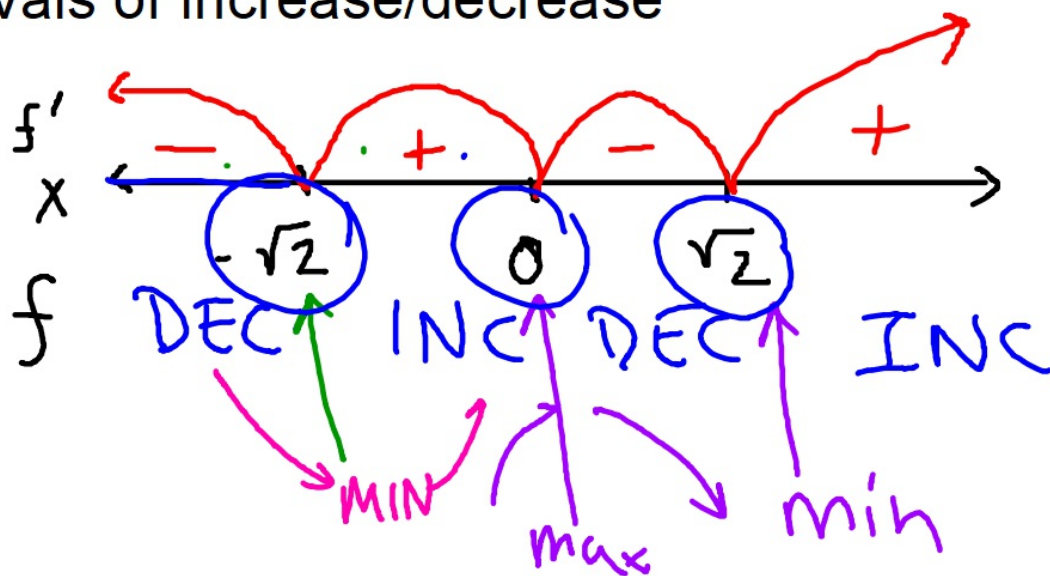
Set  $y'$  equal to zero, solve; consider where it is undefined.

3. Plot C.N. on number line, do bunnyhops for signage

4. A sign change must occur for a max or min.

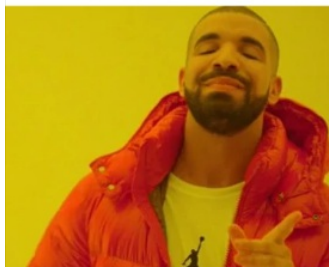
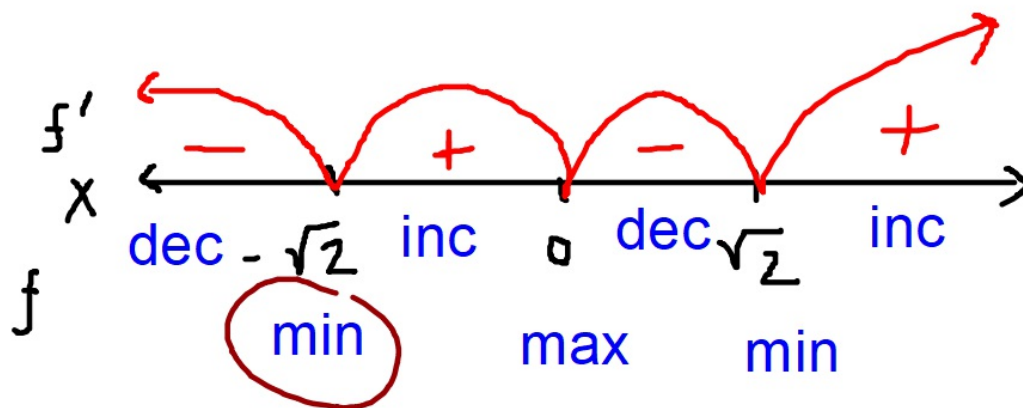
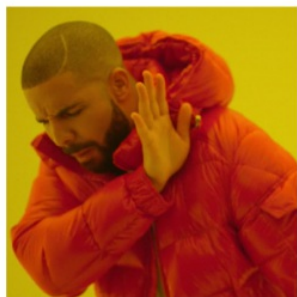
This sign chart tells you 2 things:

- locations of extrema
- intervals of increase/decrease



A sign chart is not sufficient for getting credit on assessment/AP test

Must explain verbally!



"F has a relative maximum at  $x=0$  BECAUSE  
F' changes sign from positive to negative"

f has a rel min @  $x = -\sqrt{2}$  and  $x = \sqrt{2}$   
BECAUSE  $F'$  changes from  $- \rightarrow +$ .

Find the x-coordinate(s) where  $y = x^3 - 2x^2 - 1$  has relative extrema. Justify your classifications.

$$\frac{dy}{dx} = 3x^2 - 4x = 0$$

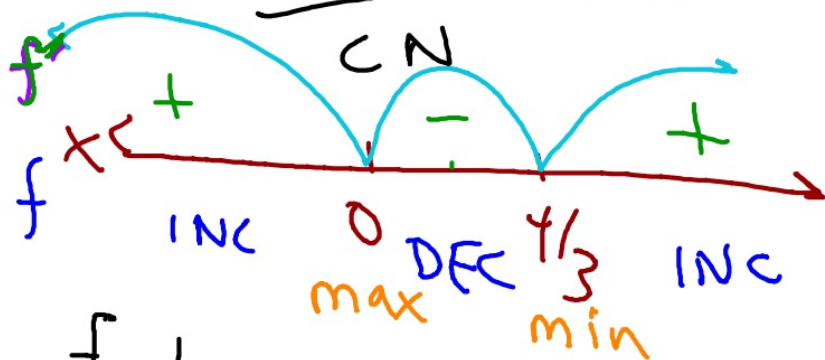
$$x(3x - 4) = 0$$

$$x = 0 \quad x = \frac{4}{3}$$

① Find C.N.

② Plot on Sign Chart

③ Pick (test values, plug into  $f'$ )



$f$  has rel max @  $x = 0$  b/c  $f'$  changes

$f$  has rel min @  $x = \frac{4}{3}$  b/c  $f'$  changes from  $-$  to  $+$

Practice...

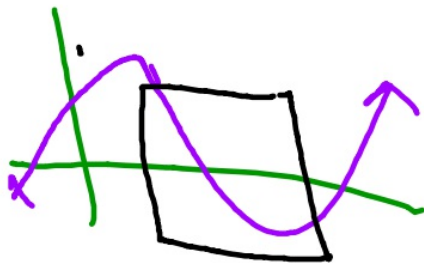
Kahoot!



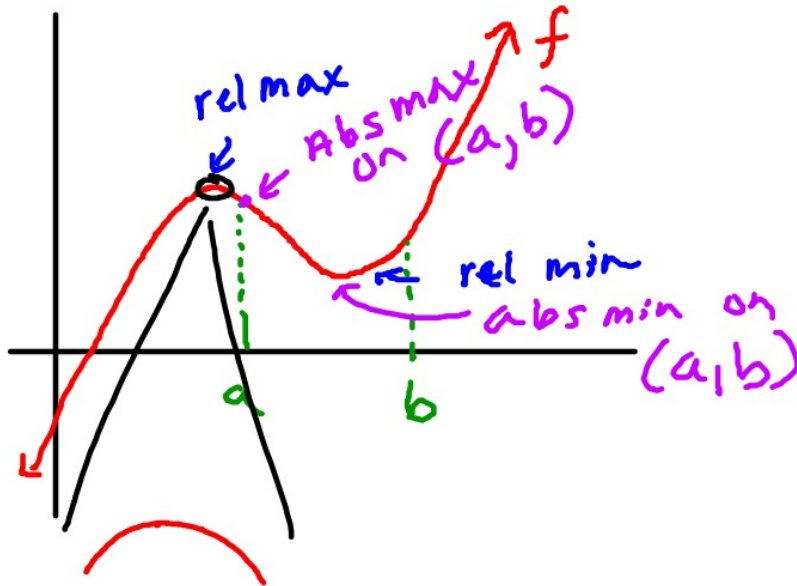
## Absolute Extrema...a bit of a misnomer

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- (Global max/min)
- must occur on an interval



## Absolute (Global) Extrema



Absolute Extremes  
occur on intervals

They occur at relative ext.  
OR at endpoints!



Every function continuous on an interval has an absolute max and absolute min (extreme value theorem)

it either occurs at an endpoint, or at a relative max/min

How to find absolute extrema over an interval

$$g(t) = 2t^3 + 3t^2 - 12t + 4 \text{ on the interval } [-4, 2]$$

$$g'(t) = 6t^2 + 6t - 12 = 0$$

$$6(t^2 + t - 2) = 0$$

$$6(t+2)(t-1) = 0$$

$$t = -2, t = 1$$

$$g(-4) = 2(-4)^3 + 3(-4)^2 - 12(-4) + 4 = -28$$

$$g(2) = 2(2)^3 + 3(2)^2 - 12(2) + 4 = 8$$

$$g(1) = 2(1)^3 + 3 - 12 + 4 = -3$$

$$g(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 4 = 24$$

① Find C.N.

② Plug endpoints and any C.N. into  $f$

③ Abs max: biggest output

Abs min: smallest output

Abs max	min
@ $x = -2$	@ $x = 1$

Find the x-coordinates where  $f(x)$  has an absolute max and absolute min on  $[-3, 1]$  for  $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$

$$f' = x^3 - x^2 - 6x = 0$$

$$x(x^2 - x - 6) = 0$$

$$x(x-3)(x+2) = 0$$

$x=0$   
 ~~$x=3$~~  not in interval  
 $x=-2$

$$f(0) = 0$$

$$f(-2) = \frac{-10\frac{2}{3}}{2\frac{1}{4}}$$

$$f(-3) = \frac{1}{4} - \frac{1}{3} - 3$$

$$f(1) = -\frac{1}{12} - 3 = -3\frac{1}{12}$$

Max  
 @  $x = -3$   
 Min  
 @  $x = -2$

## Skills on assessment

D-AD7: interpreting an  $F'$  graph for inc/dec and rel max/min

D-AD8: finding relative and absolute extrema algebraically

D-AD9: finding intervals of inc/dec algebraically

D-AD5: implicit derivatives

D-DC2: Interpreting derivatives