

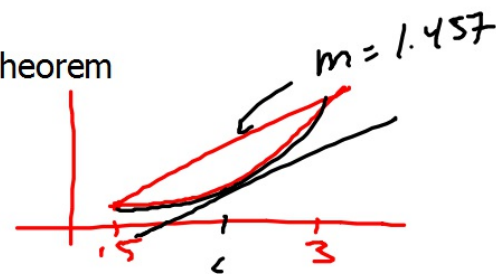
Good afternoon: warm up in notebooks

Find the value of c guaranteed to exist by the Mean Value Theorem
for $f(x) = x \ln(x)$ on the interval $[0.5, 3]$

a b

Avg. Rate

$$\frac{f(3) - f(0.5)}{3 - 0.5} = \frac{3 \ln 3 - \frac{1}{2} \ln \frac{1}{2}}{2.5} = 1.457$$



Instant. Rate

$$f'(x) = 1 \cdot \ln(x) + x \cdot \frac{1}{x}$$

$$f'(x) = \ln(x) + 1$$

$$\ln(x) + 1 = 1.457$$

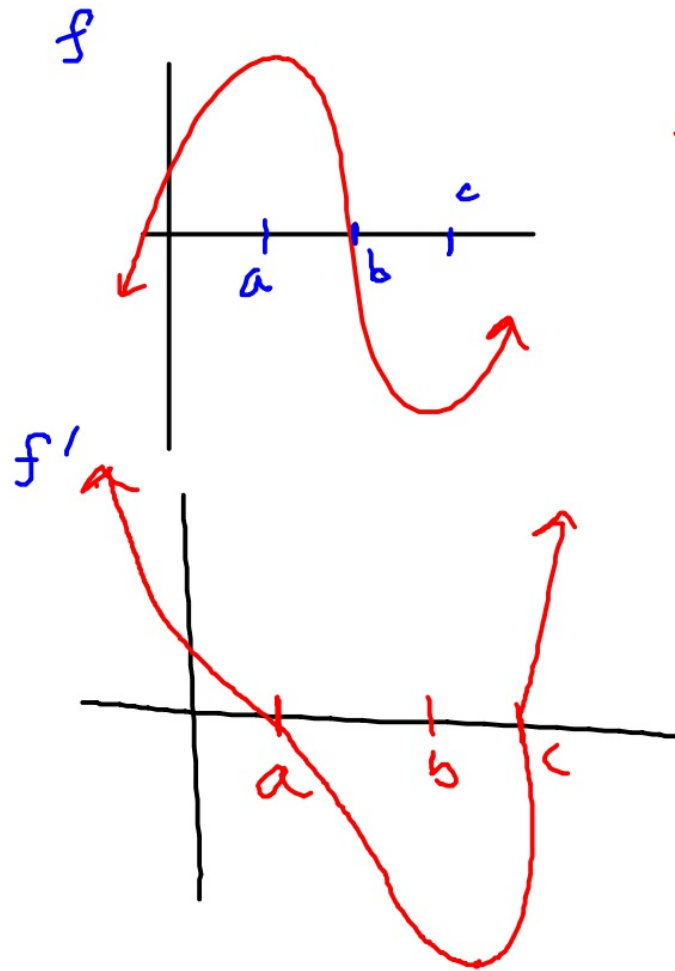
$$\ln(x) = 0.457$$
$$\log_e x = 0.457$$

$$\log_b x = a$$
$$\updownarrow$$
$$b^a = x$$

$$e^{0.457} = x$$
$$1.579 \approx x$$

bit.ly/xlnxmvt

Intervals of increase and decrease

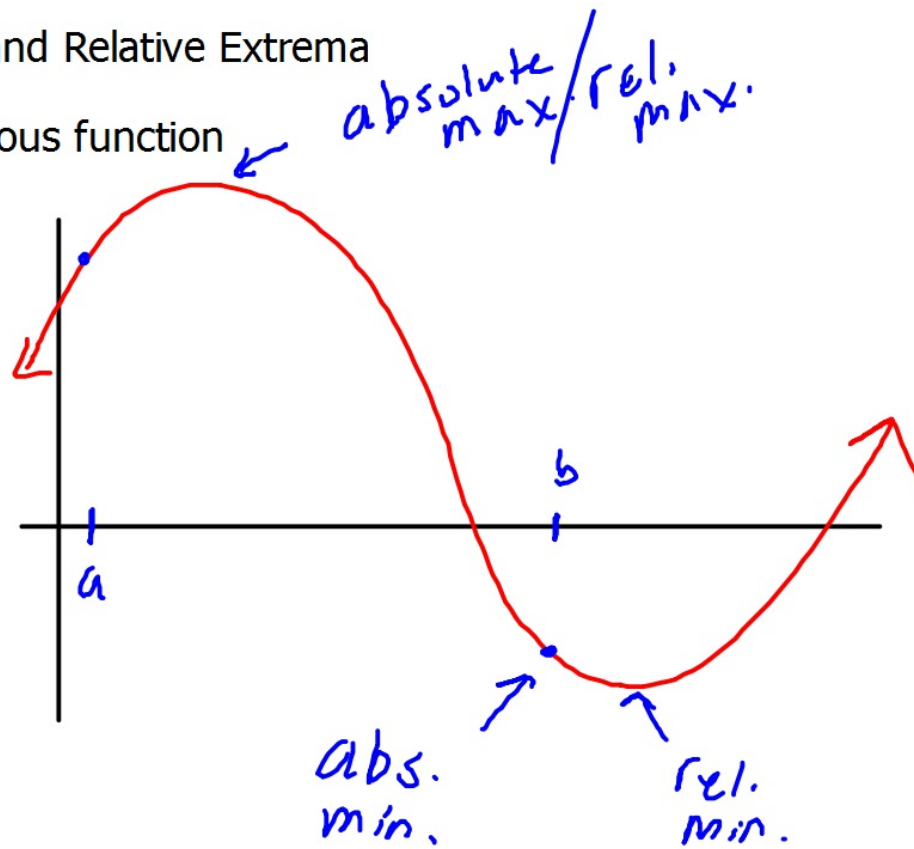


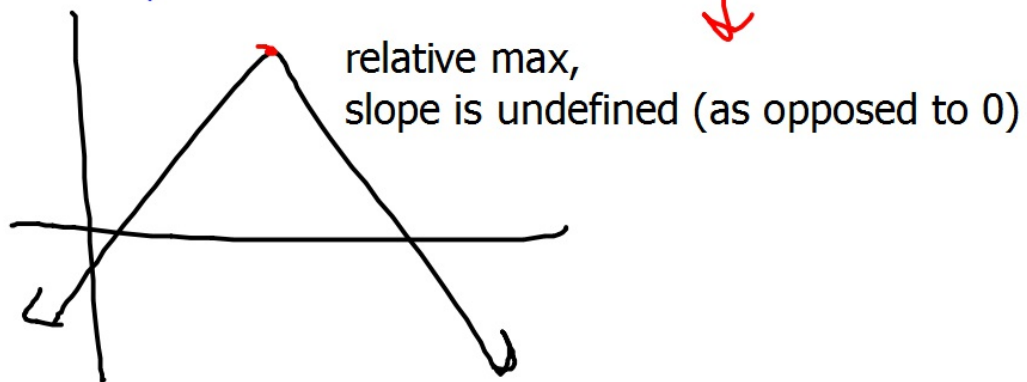
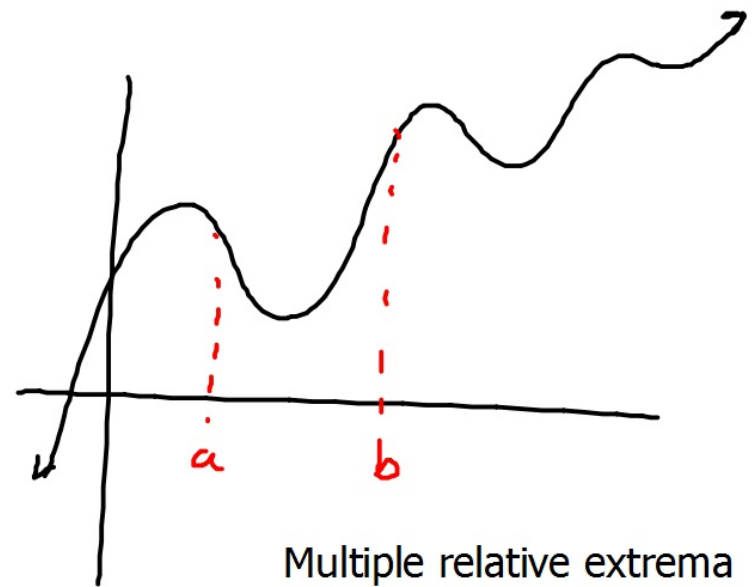
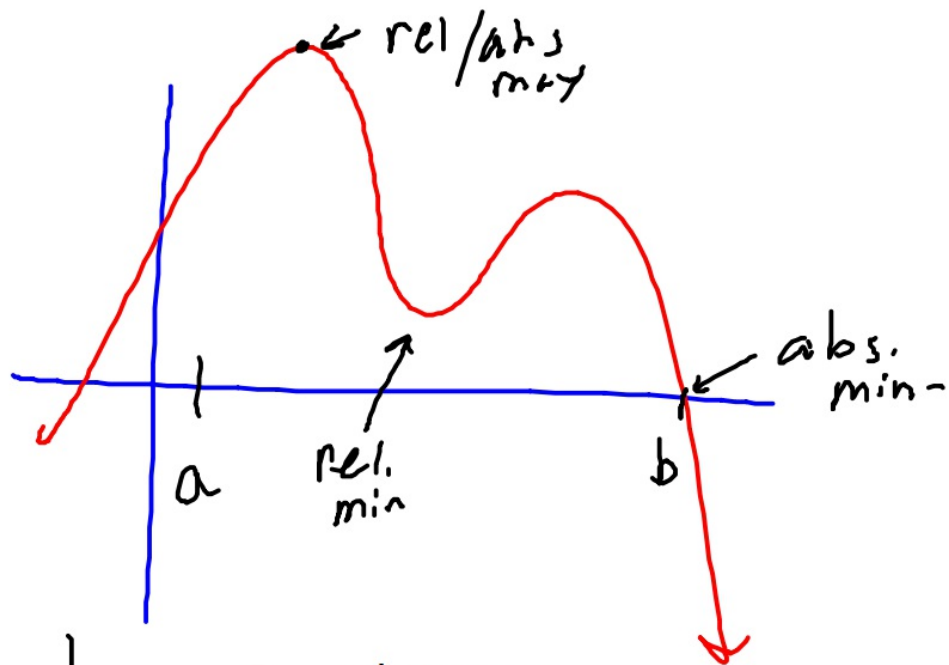
f increasing: $(-\infty, a)$ and (c, ∞)
 $f' > 0$ on "

f decreasing (a, c)
 $f' < 0$ " "

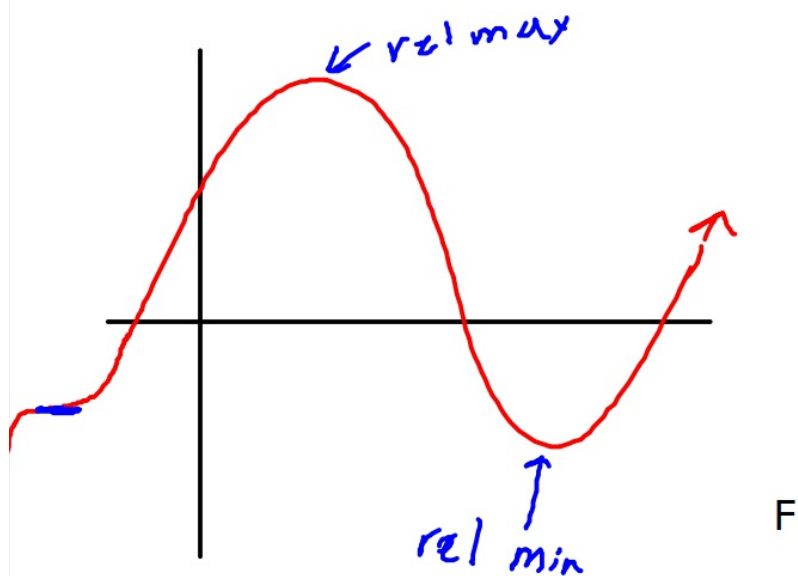
Absolute and Relative Extrema

Continuous function



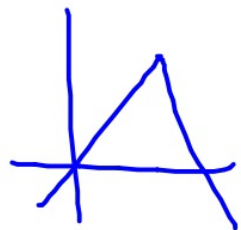


Where do maxima and minima occur?



Absolute (Global) Extrema
vs
Relative (Local) Extrema

• Rel max/min
occur when
 $f' = 0$ or f' undefined.



• Abs max/min
occur either @
rel. max/min
or at endpoints

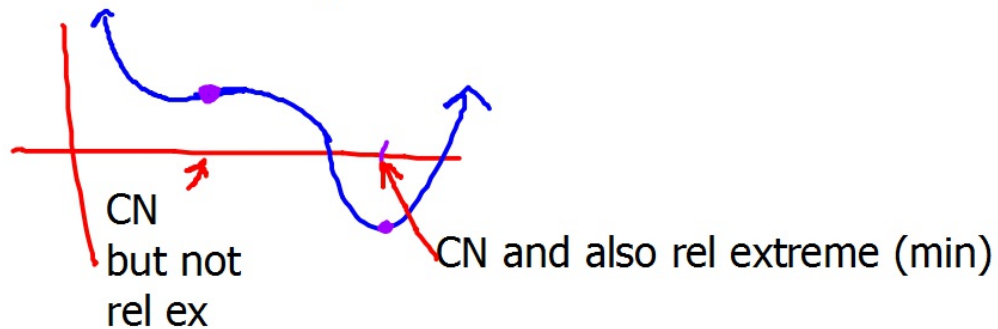
Definition:

c is a critical number of $f(x)$ if $f'(c) = 0$ OR if $f'(c)$ is undefined

All relative extremes occur at critical numbers

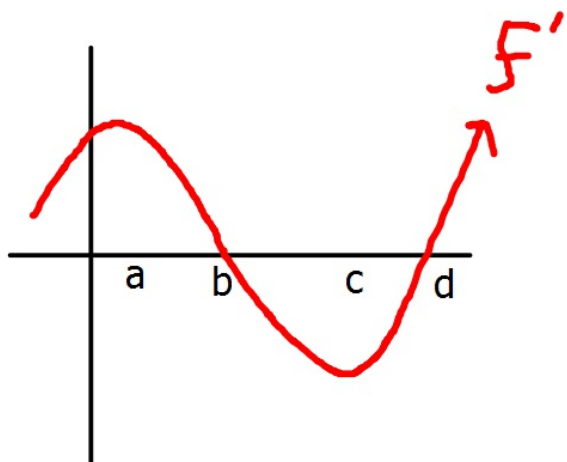
BUT

not all critical numbers yield relative extremes



Enter in your PIN

Where does F have a relative maximum?

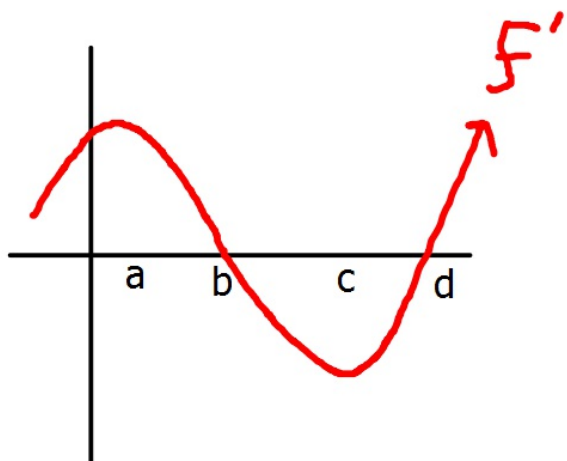


- A a only
- B b only
- C c only
- D d only
- E a and c
- F a and b

Answer:

B

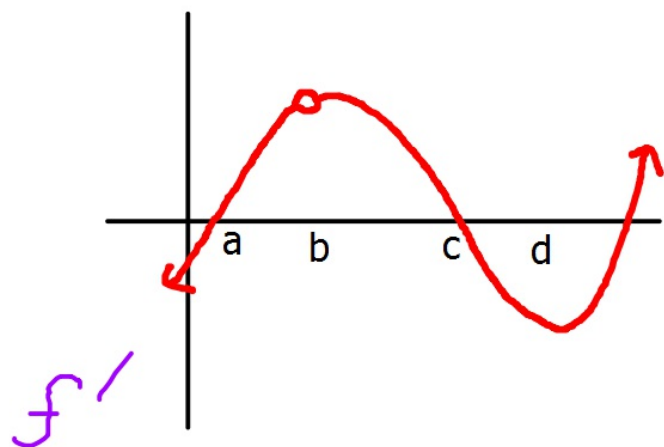
Where does F have critical numbers?



- A a only
- B b only
- C c only
- D d only
- E a and c
- F b and d

Answer: F

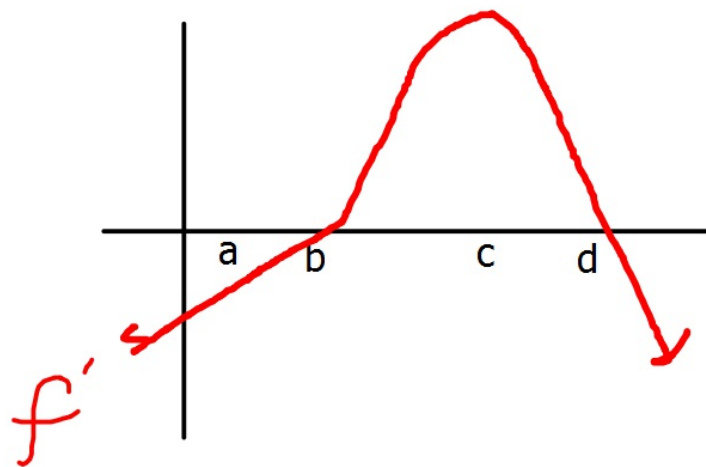
Where does F have a critical number but NOT a relative extrema?



- A a only
- B b only
- C c only
- D d only

Answer
B

Where does F have a relative minimum?

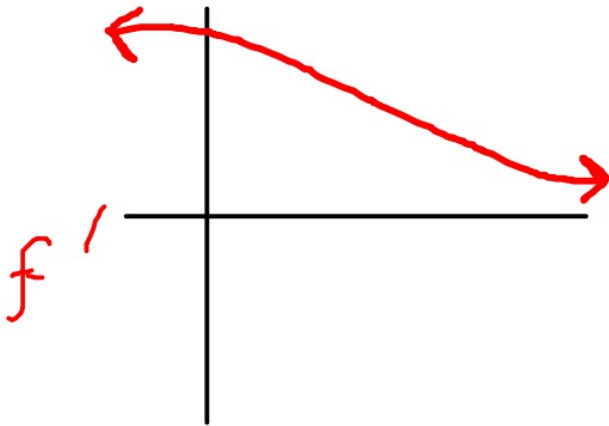


- A a only
- B b only
- C c only
- D d only
- E a and c
- F a and b

Answer

B (f' goes from negative to positive)

Which is true about F?



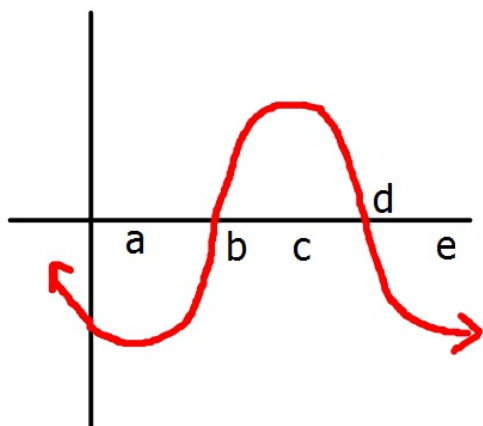
- A always increasing
- B always decreasing
- C increases, then decreases
- D decreases, then increases

Answer

A

since f' is always positive, f is always increasing

Where does F have a relative maximum?



A a only

B b only

C c only

D d only

E e only

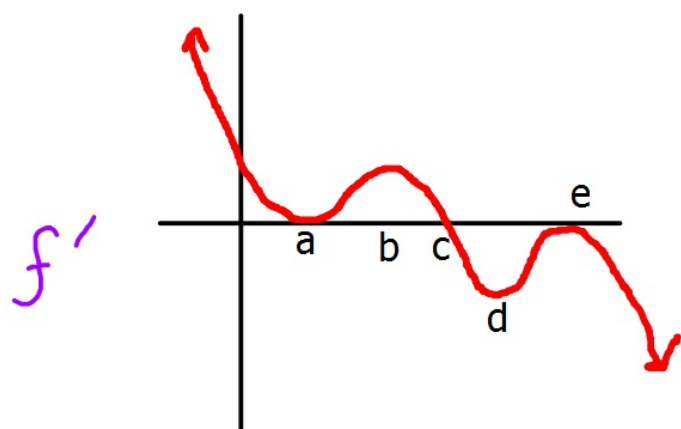
F both a and d

Answer

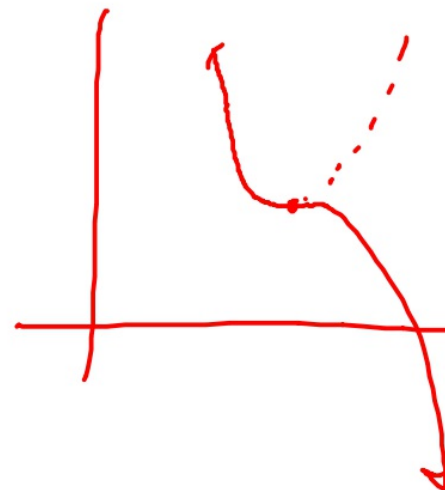
D

f' changes from positive to negative

Where does F have a relative minimum?



- A a only
- B b only
- C c only
- D d only
- E e only



Answer:

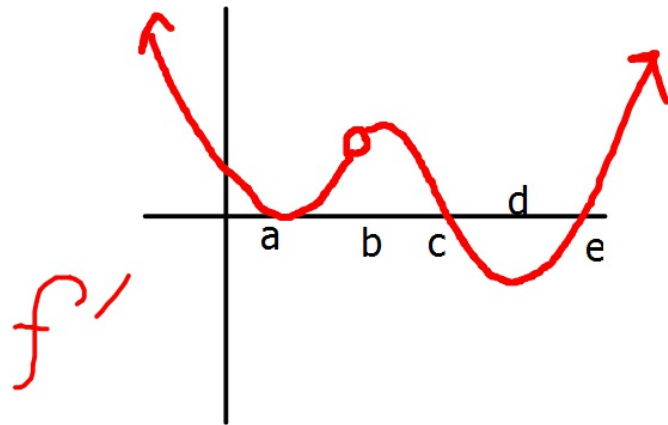
none of the above -_-

sorry forgot to make that a choice

A and E are critical numbers, but no sign change

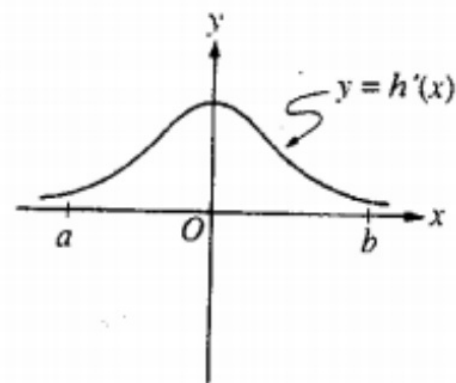
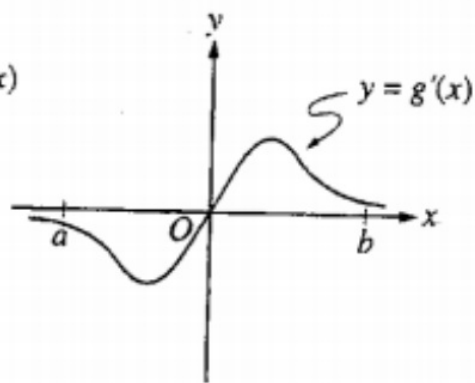
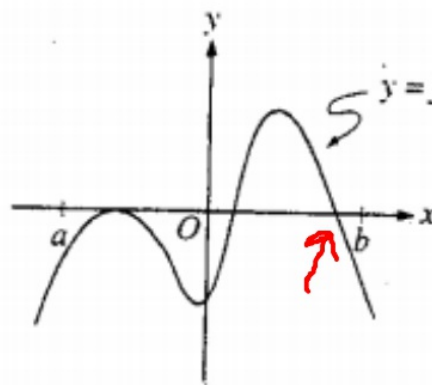
C represents a relative maximum (pos to neg)

Which is NOT a critical number of F ?



- A a only
- B b only
- C c only
- D d only
- E e only

Answer
D

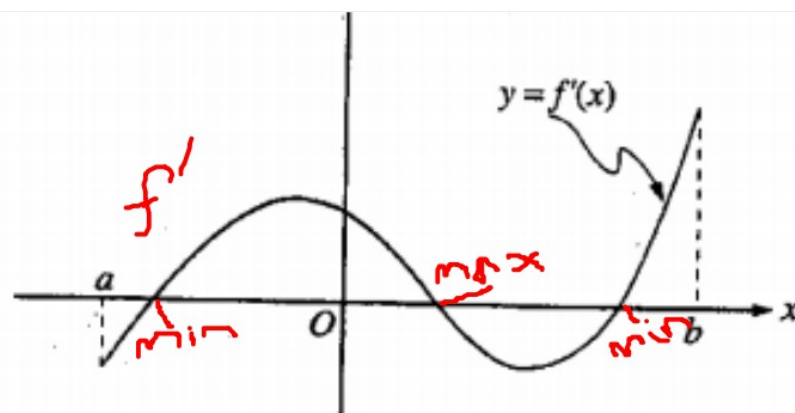


- 78 79. The graphs of the derivatives of the functions f , g , and h are shown above. Which of the functions f , g , or h have a relative maximum on the open interval $a < x < b$?

- (A) f only
- (B) g only
- (C) h only
- (D) f and g only
- (E) f , g , and h

Answer

A



12. The graph of f' , the derivative of f , is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b) ?
- (A) One relative maximum and two relative minima
 - (B) Two relative maxima and one relative minimum
 - (C) Three relative maxima and one relative minimum
 - (D) One relative maximum and three relative minima
 - (E) Three relative maxima and two relative minima

Find all critical numbers of the function

$$f(x) = 4x / (x^2 + 1)$$

$$f(x) = \frac{4x}{x^2 + 1} \Rightarrow f'(x) = \frac{4(x^2 + 1) - 4x \cdot 2x}{(x^2 + 1)^2}$$

$$\frac{4x^2 + 4 - 8x^2}{(x^2 + 1)^2}$$

$$f' = \frac{4 - 4x^2}{(x^2 + 1)^2} \rightarrow 4 - 4x^2 = 0$$

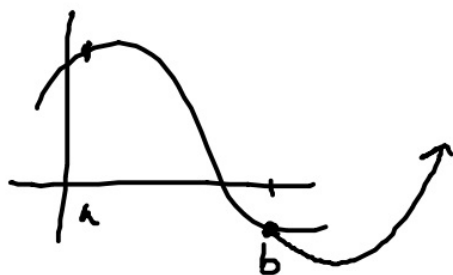
$$\begin{aligned} 4 &= 4x^2 \\ \sqrt{4} &= \sqrt{4x^2} \\ \pm 2 &= \pm 2x \\ \pm 1 &= x \end{aligned}$$

$$\begin{aligned} \cancel{4} - \cancel{4}x^2 &= 0 \\ 4(1 - x^2) &\rightarrow 4(1 + x)(1 - x) = 0 \end{aligned}$$

$$\boxed{x = \pm 1}$$

How to find absolute extrema over an interval

$$g(t) = 2t^3 + 3t^2 - 12t + 4 \text{ on the interval } [-4, 2]$$



① Find critical #'s

② Plug C.N. & Endpts
into f .

Example on

p. 165

Biggest output \rightarrow Abs max

Smallest output \rightarrow Abs min

How to find absolute extrema over an interval

$g(t) = 2t^3 + 3t^2 - 12t + 4$ on the interval $[-4, 2]$

Abs. extrema
D.C.N. or
C.N. or
Endpts.

① find critical Numbers

$$g'(t) = 6t^2 + 6t - 12 = 0$$

$$6(t^2 + t - 2) = 0$$

$$6(t-1)(t+2) = 0$$

$$t-1=0 \quad t+2=0$$

$$t=1 \quad t=-2$$

Critical Numbers.

② Plug C.N. and
endpts into function:

$$\text{C.N. } \begin{cases} g(1) = 2 + 3 - 12 + 4 = -3 \\ g(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 4 \\ \quad \quad \quad 16 + 12 + 24 + 4 \\ \quad \quad \quad = 56 \end{cases}$$

$$\text{E.P. } \begin{cases} g(-4) = 2(-4)^3 + 3(-4)^2 - 12(-4) + 4 \\ \quad \quad \quad -128 + 48 + 48 + 4 \\ \quad \quad \quad = -28 \\ g(2) = 2(2)^3 + 3(2)^2 - 12(2) + 4 \\ \quad \quad \quad 16 + 12 - 24 + 4 \\ \quad \quad \quad = 8 \end{cases}$$

Biggest is 56 (@ $t = -2$)

Smallest is -28 (@ $t = -4$)

Abs max: $(-2, 56)$

Abs min: $(-4, -28)$

Homework

p.167 #17-28