Good afternoon: warm up is continuous (😂) from yesterday

For $2x^3-3y^2=8$, show that $y'' = \dfrac{2xy^2-x^4}{y^3}$

$6x^2 - 6yy' = 0$

$y' = \dfrac{-6x^2}{-6y} = \dfrac{x^2}{y}$ ← f, ← g

f: $x^2$  g: $y$

f': $2x$  g': $y'$

$y'' = \dfrac{2xy - x^2 \cdot y'}{y^2}$

sub.

$\left[\dfrac{2xy - x^2 \cdot \frac{x^2}{y}}{y^2}\right] \cdot \dfrac{y}{y}$

$\dfrac{2xy^2 - x^4}{y^3}$

Next assessment: Monday

tomorrow DS: retakes

vrg

Retakes ¯_(ツ)_/¯

can be open note (per skill)
max grade possible is 3 (86)

Consider the curve given by $xy^2 - x^3y = 6$.

2000 AB #4

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose $x$-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the $x$-coordinate of each point on the curve where the tangent line is vertical.

a) $1 \cdot y^2 + 2xyy' - 3x^2y - x^3y' = 0$

$2xyy' - 3x^2y - x^3y' = 3x^2y - y^2$

$y'(2xy - x^3) = 3x^2y - y^2$

$y' = \frac{3x^2y - y^2}{2xy - x^3}$

b) $x = 1 \rightarrow$ what's $y$?

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3 \quad y = -2$

$(1,3) \quad (1,-2)$

$m @ (1,-2)$: $\frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

$m @ (1,3)$: $\frac{9-9}{6-1} = \frac{0}{5} = 0$

$y - 3 = m(x-1)$

$y + 2 = m(x-1)$

$y = 3$

$y + 2 = 2(x-1)$

* $2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x = 0$

$2y - x^2 = 0$

$2y = x^2$

$y = \frac{1}{2}x^2$

$xy^2 - x^3y = 6$

$x \cdot \frac{1}{4}x^4 - x^3 \cdot \frac{1}{2}x^2 = 6$

$\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$-\frac{1}{4}x^5 = 6$

$x^5 = -24$

$x = \sqrt[5]{-24}$

# Let's talk AP test

May 14, 2019

50% of score: multiple choice

(45) 30Q 60min no calculator

15Q 45min calculator

50% of score: free response

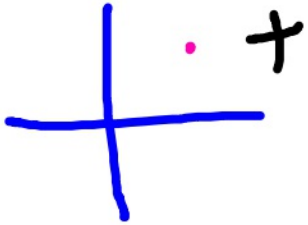
54 2Q, 30min calculator

4Q, 60min, no calculator

15 min/q.

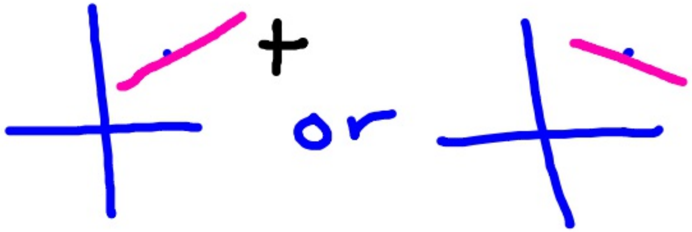
What does y tell you?

where to go, a point in space for
a particular x



What does y' tell you?

increasing or decreasing
as you pass through that point

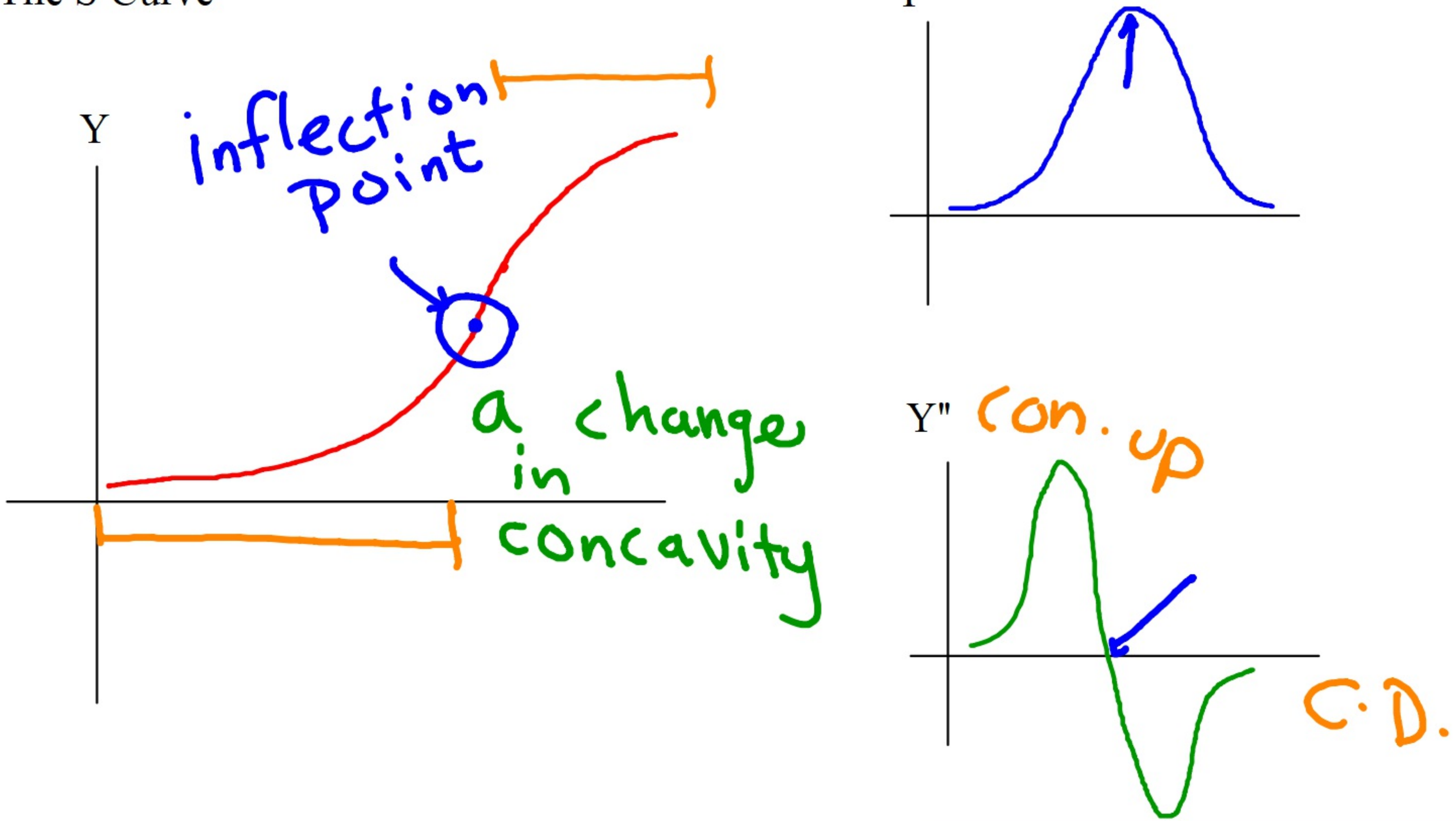


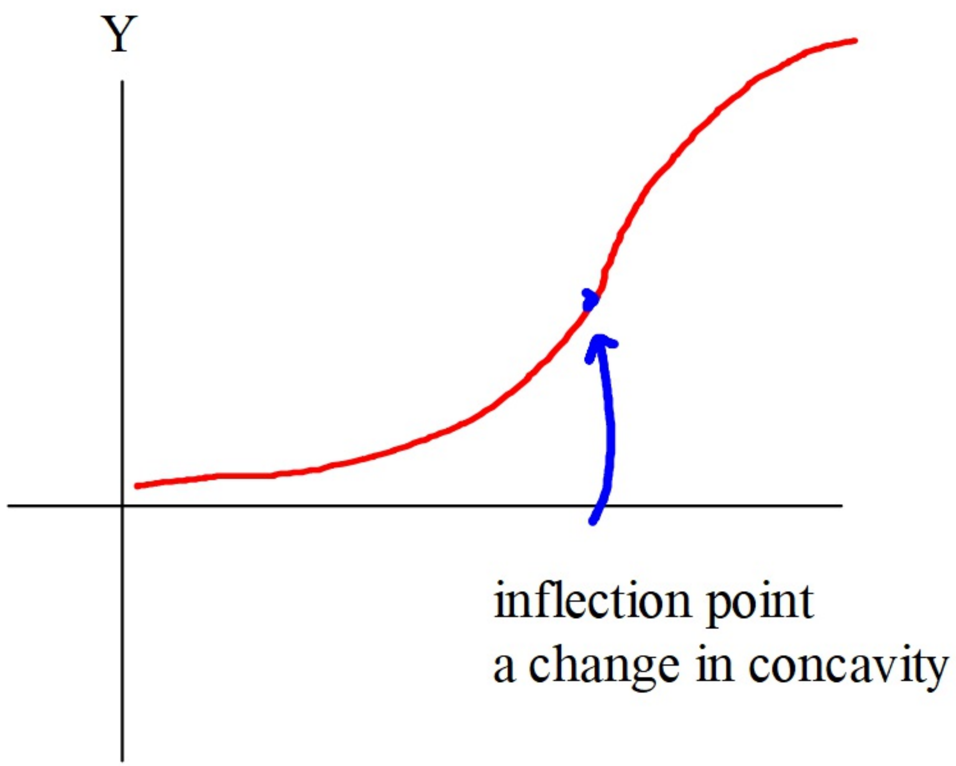


What does y'' tell you?

increasingly or decreasingly
increasing or decreasing

The S Curve





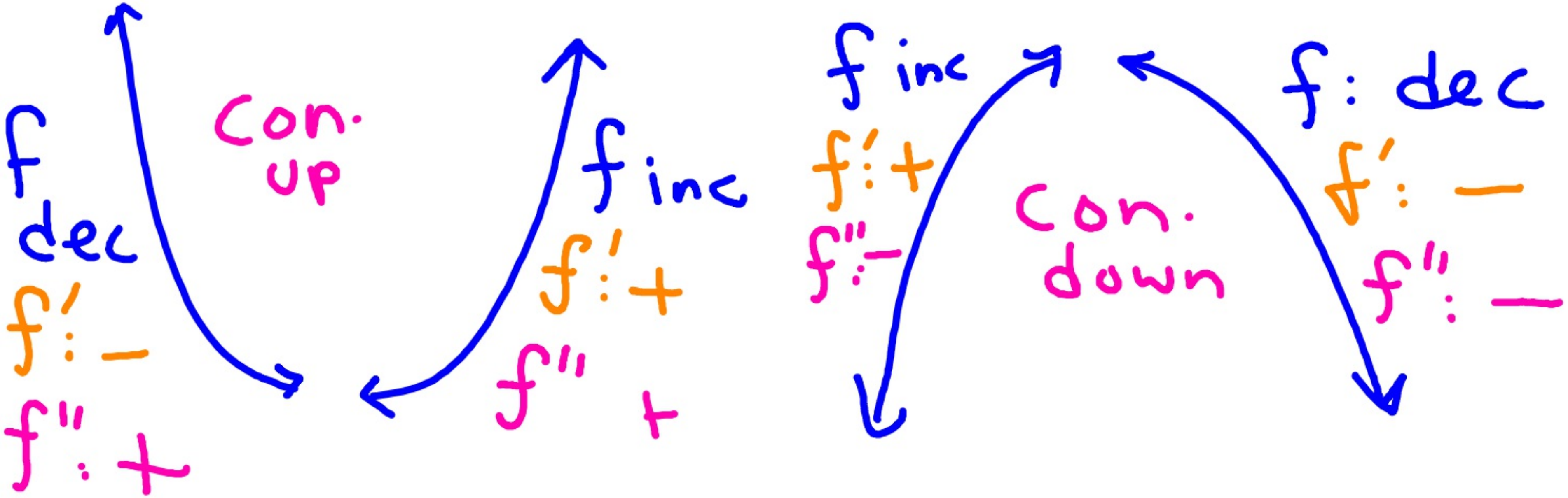

What might this model?

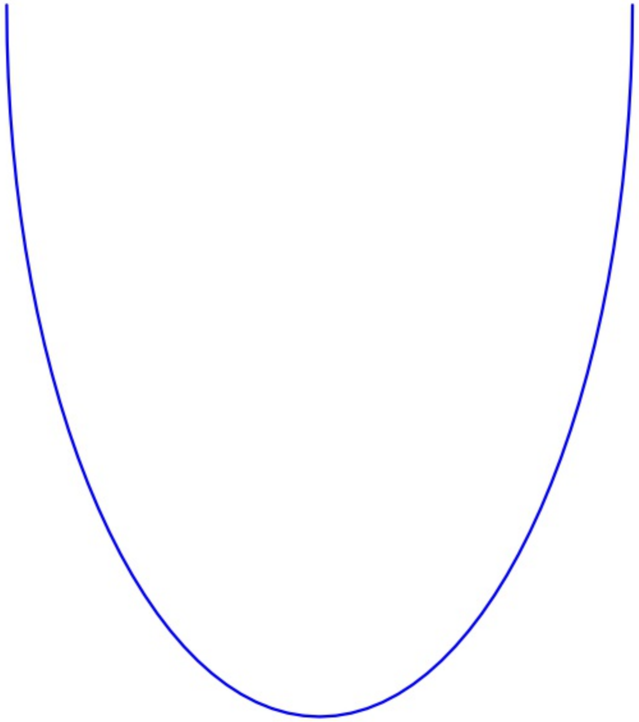
- new toy demand
- bacteria growth
- Mozart's symphony production
- airline traffic
- ???

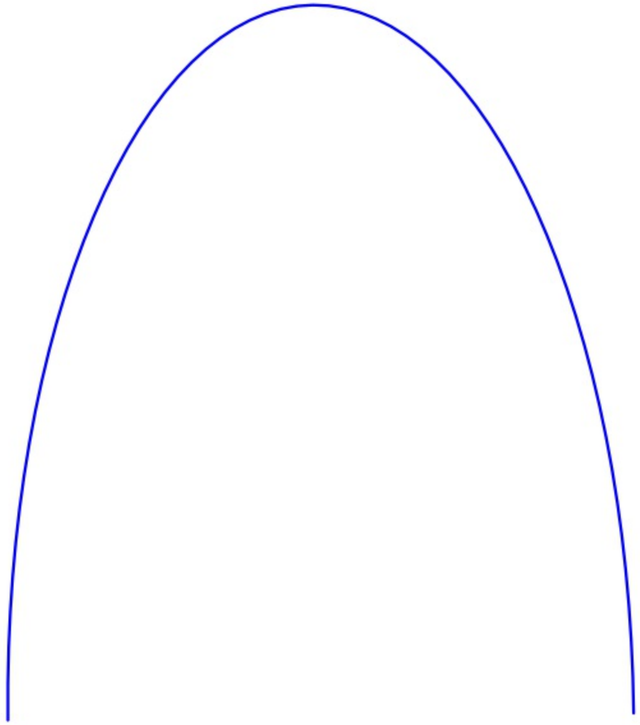
# Concavity and the 4 kinds of curvature

parabolæ

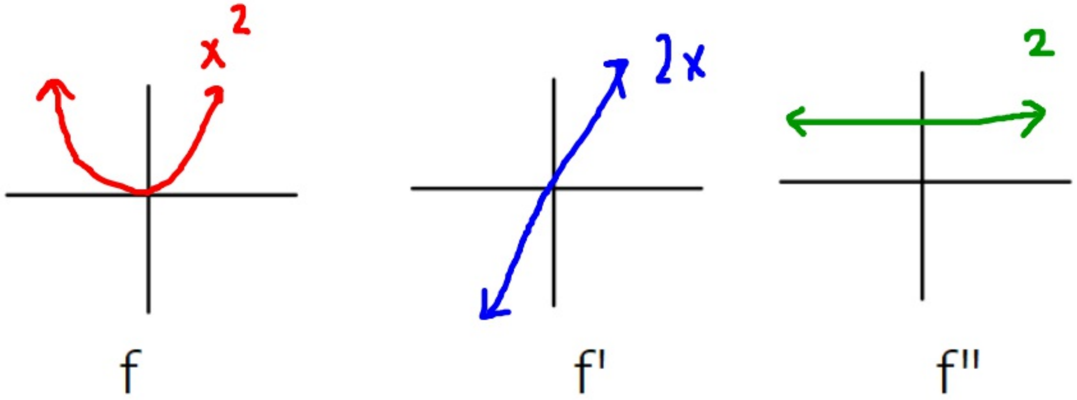
| | | | |
|---|---|---|---|
| concave up: | holds water | f ' increasing | f " > 0 |
| concave down: | doesn't | f ' decreasing | f " < 0 |



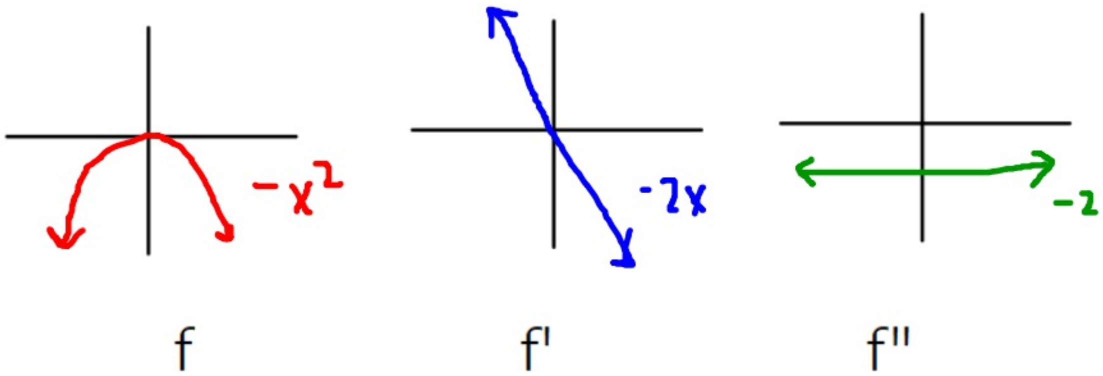




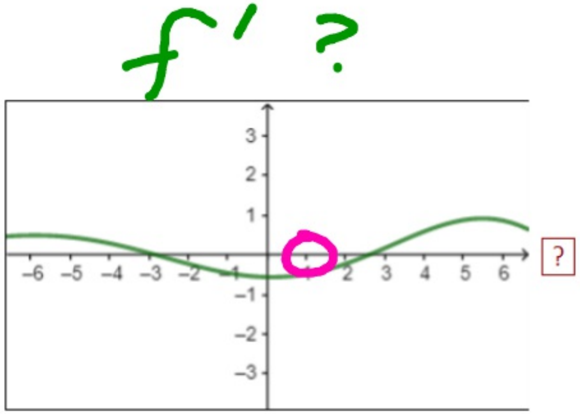
# Concave up



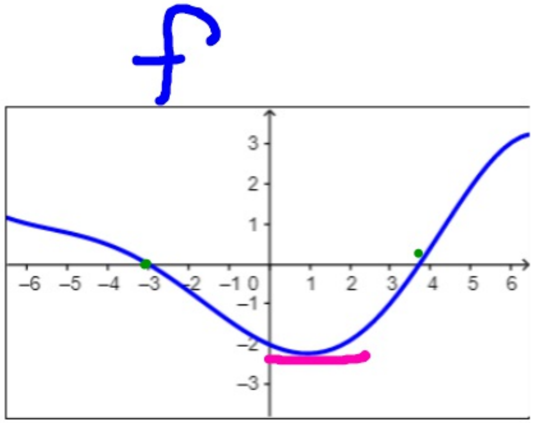

# Concave down

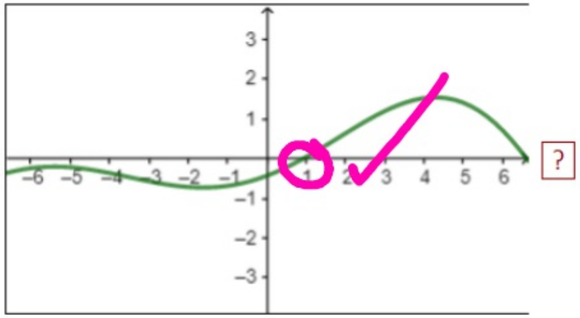


## Identify the Derivative Function




1 finger (not that one)



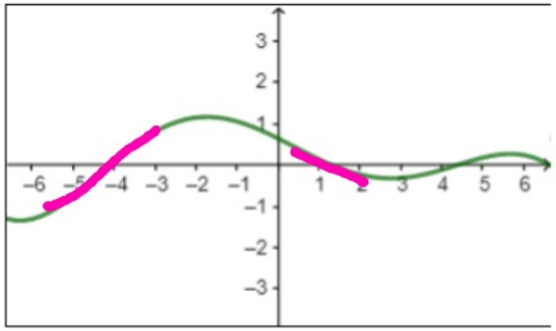



2 fingers

Reset Graphs

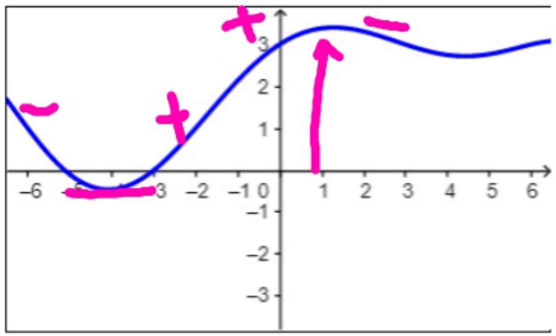
3 fingers

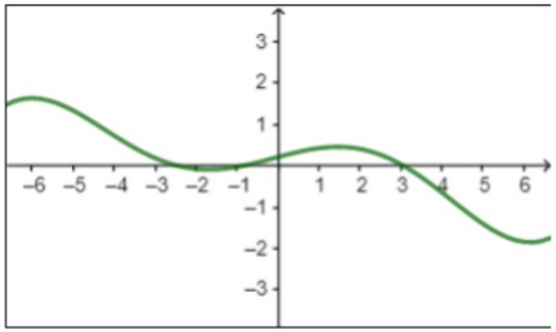
## Identify the Derivative Function



?

1 finger

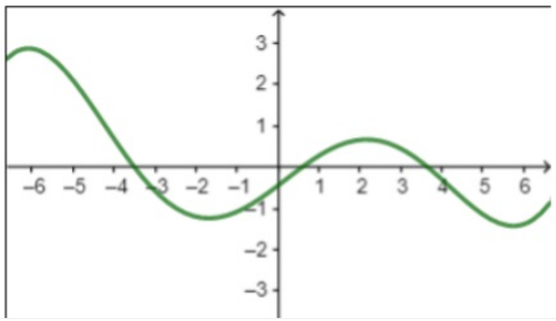




?

2 fingers

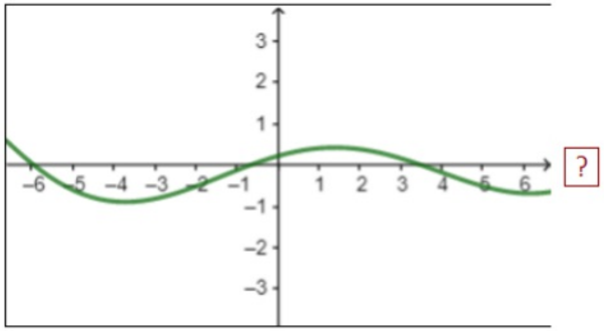
Reset Graphs



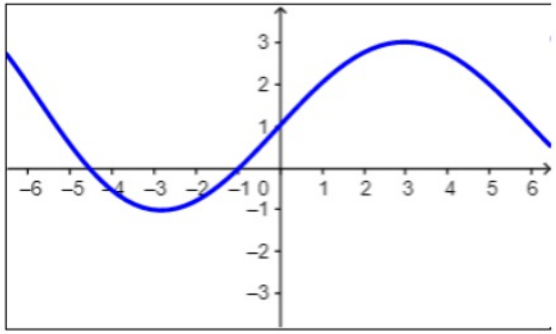
?

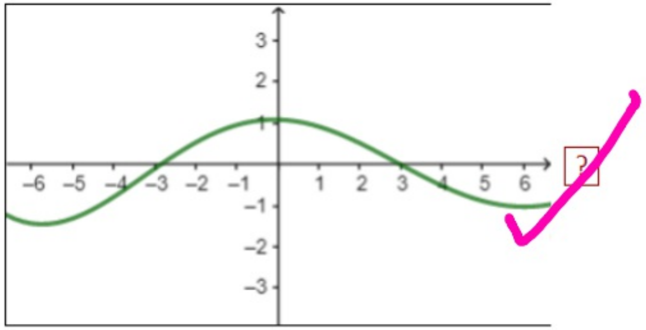
3 fingers

## Identify the Derivative Function



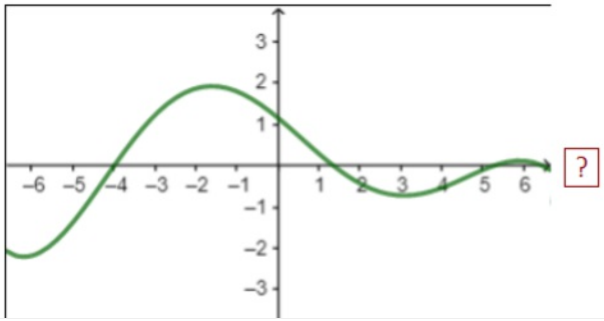
1 finger





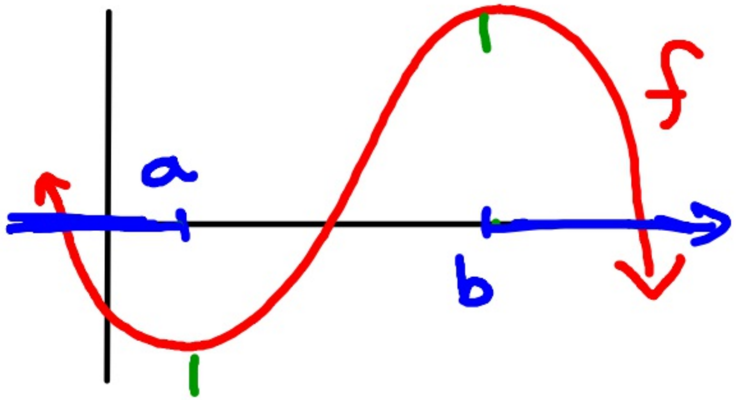
2 fingers

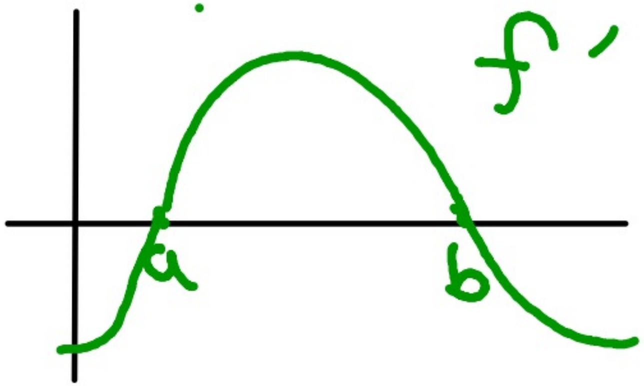
Reset Graphs



3 fingers

Intervals of Increase and Decrease




f is increasing on $(a, b)$ because $f' > 0$ there.

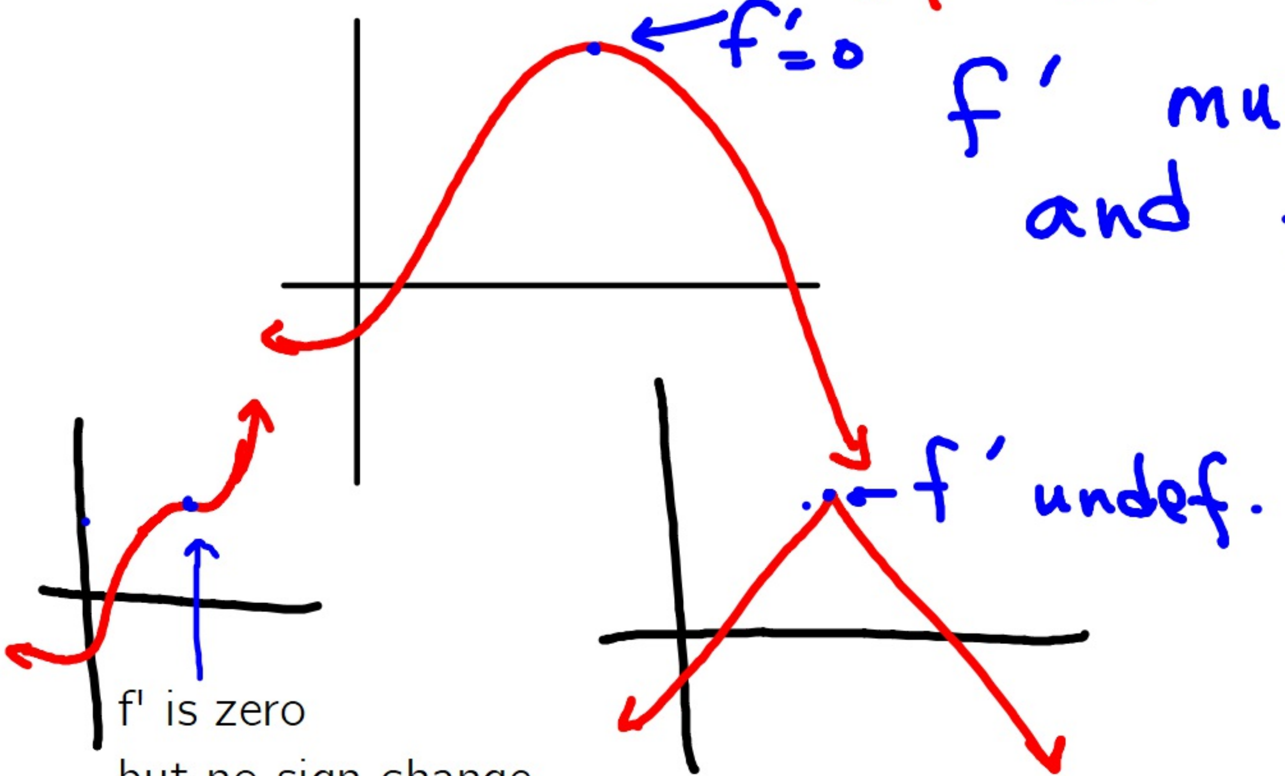
f is dec. on $(-\infty, a) \cup (b, \infty)$ b/c $f' < 0$ there.

# Relative (Local) Extrema

What does it mean to be a local maximum? (peak)

f' must be 0* (or undef.)

and f' changes sign + → −.




f' is zero
but no sign change.
slope is positive, then zero,
then positive. Not a max
or min.

Local Max and Local Min

A continuous function F has a local max at c if F' changes from pos to neg at c
" " " local min at c if F' changes from neg to pos at c

To go from positive to negative at x=c
what could be true of f '(c) ?

Definition:

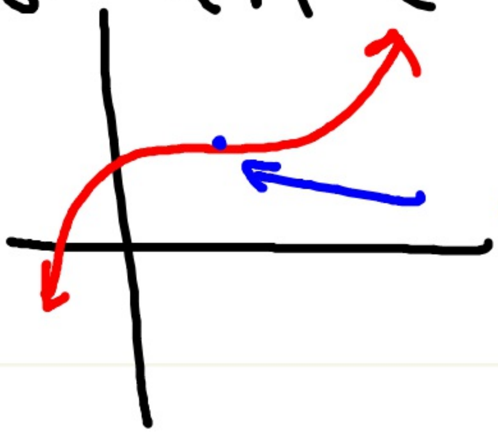
A number c, in the domain of f, is a <u>critical number</u> of f

iff f '(c) = 0 or f '(c) is undefined

$\Leftrightarrow$

**all relative extrema (max/min) occur at critical numbers

BUT not all C.N. are max/mins.



C.N. but not max/min.

How to find where a function has relative extrema

1. Take the derivative of y, y'
2. Find C.N.
   Set y' equal to zero, solve; consider where it is undefined.
3. Plot C.N. on number line, do bunnyhops for signage
4. A sign change <u>must</u> occur for a max or min.

Find the x-coordinate(s) where $y = x^4 - 4x^2 + 1$ has relative extrema. Justify your classifications.

$y' = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

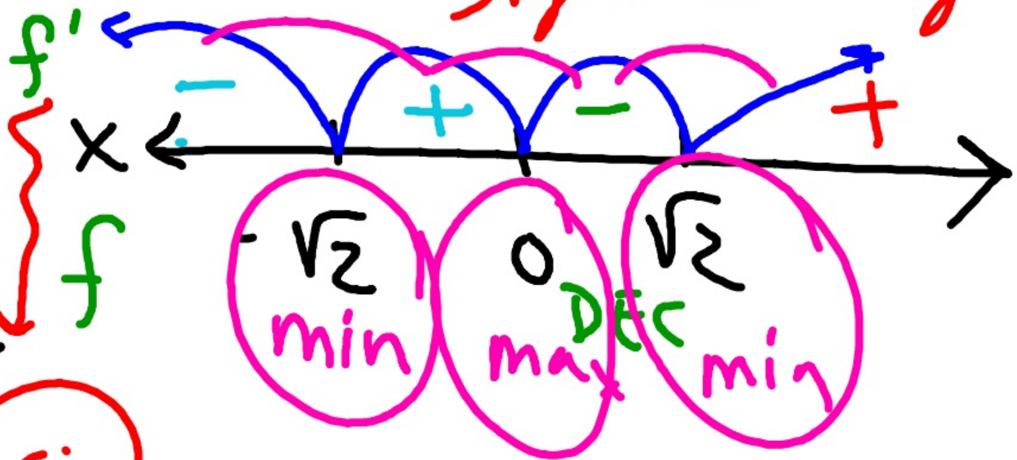
$x = 0$ $\qquad$ $x^2 = 2$

$x = \pm\sqrt{2}$

C.N.

① Find C.N.

② Look for sign changes




Where do those signs come from?

Pick numbers in each interval. Plug into f'. Find sign of result.

$f'(1) = -4$

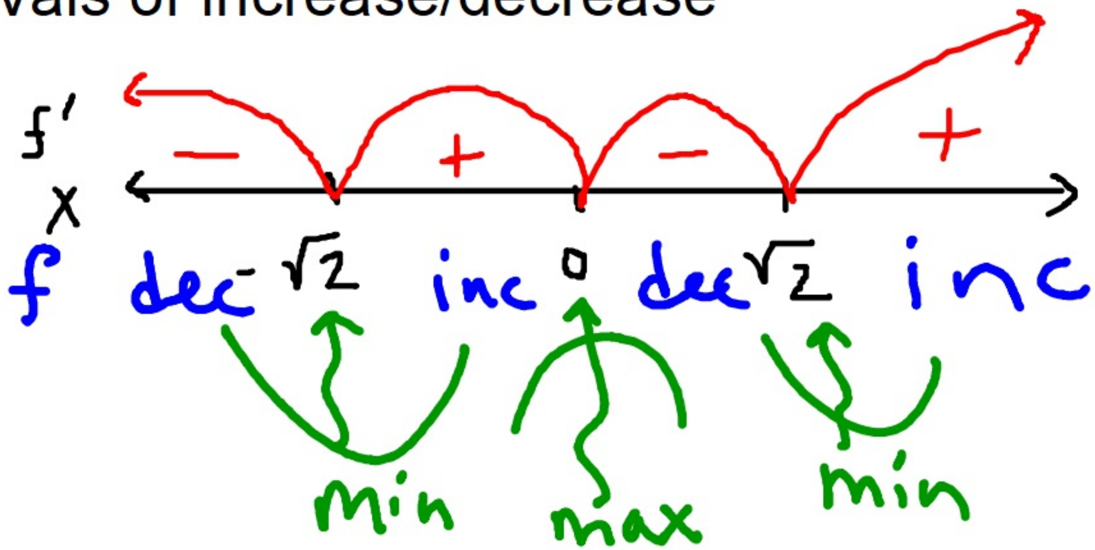
$f'(-1) = 4$

$f'(1000) = +$

$f'(-1000) = -$

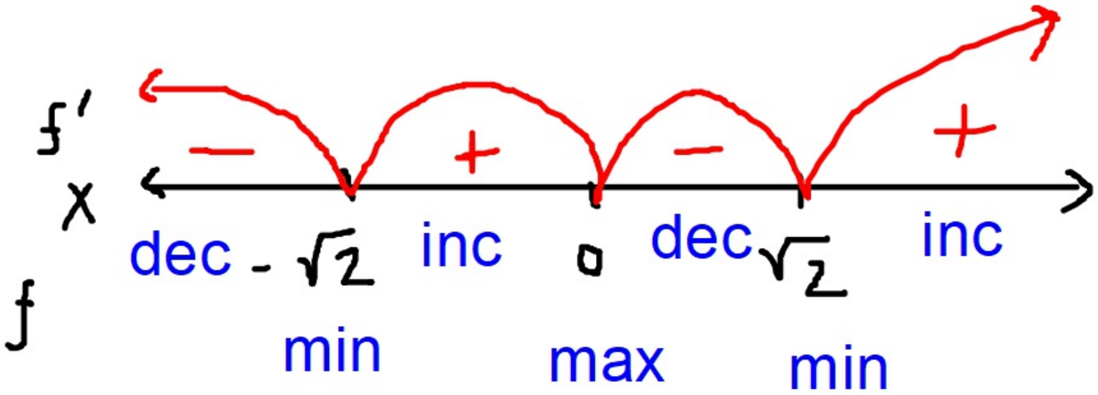
This sign chart tells you 2 things:

- locations of extrema
- intervals of increase/decrease



A sign chart is not sufficient for getting credit on assessment/AP test

Must explain verbally!




[redacted]

Find the x-coordinate(s) where $y= x^3-2x^2-1$ has relative extrema. Justify your classifications.

# Practice…

HW:
handout