15. $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6(x - 2) = 0$ when $x = 2$.

Concave upward: $(2, \infty)$

Concave downward: $(-\infty, 2)$

Point of inflection: $(2, 8)$

16. $f(x) = -x^3 + 6x^2 - 5$

$f'(x) = -3x^2 + 12x$

$f''(x) = -6x + 12 = -6(x - 2) = 0$ when $x = 2$.

Concave upward: $(-\infty, 2)$

Concave downward: $(2, \infty)$

Point of inflection: $(2, 11)$

17. $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x + 2)$

$f''(x) = 0$ when $x = 0, -2$

Concave upward: $(-\infty, -2), (0, \infty)$

Concave downward: $(-2, 0)$

Points of inflection: $(-2, -8)$ and $(0, 0)$

18. $f(x) = 4 - x - 3x^4$

$f'(x) = -1 - 12x^3$

$f''(x) = -36x^2 = 0$ when $x = 0$.

Concave downward: $(-\infty, \infty)$

No points of inflection

19. $f(x) = x(x-4)^3$

$f'(x) = x\left[3(x-4)^2\right] + (x-4)^3 = (x-4)^2(4x-4)$

$f''(x) = 4(x-1)\left[2(x-4)\right] + 4(x-4)^2 = 4(x-4)\left[2(x-1) + (x-4)\right] = 4(x-4)(3x-6) = 12(x-4)(x-2)$

$f''(x) = 12(x-4)(x-2) = 0$ when $x = 2, 4$.

Intervals:	$-\infty < x < 2$	$2 < x < 4$	$4 < x < \infty$
Sign of $f''(x)$:	$f''(x) > 0$	$f''(x) < 0$	$f''(x) > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $(-\infty, 2), (4, \infty)$

Concave downward: $(2, 4)$

Points of inflection: $(2, -16), (4, 0)$

20. $f(x) = (x-2)^3(x-1)$

$f'(x) = (x-2)^2(4x-5)$

$f''(x) = 6(x-2)(2x-3)$

$f''(x) = 0$ when $x = \frac{3}{2}, 2$.

Intervals:	$-\infty < x < \frac{3}{2}$	$\frac{3}{2} < x < 2$	$2 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $\left(-\infty, \frac{3}{2}\right), (2, \infty)$

Concave downward: $\left(\frac{3}{2}, 2\right)$

Points of inflection: $\left(\frac{3}{2}, -\frac{1}{16}\right), (2, 0)$

21. $f(x) = x\sqrt{x+3}$, Domain: $[-3, \infty)$

$$f'(x) = x\left(\frac{1}{2}\right)(x+3)^{-1/2} + \sqrt{x+3} = \frac{3(x+2)}{2\sqrt{x+3}}$$

$$f''(x) = \frac{6\sqrt{x+3} - 3(x+2)(x+3)^{-1/2}}{4(x+3)}$$

$$= \frac{3(x+4)}{4(x+3)^{3/2}} = 0 \text{ when } x = -4.$$

$x = -4$ is not in the domain. f'' is not continuous at $x = -3$.

Interval:	$-3 < x < \infty$
Sign of f'':	$f'' > 0$
Conclusion:	Concave upward

Concave upward: $(-3, \infty)$

There are no points of inflection.

22. $f(x) = x\sqrt{9-x}$, Domain: $x \le 9$

$$f'(x) = \frac{3(6-x)}{2\sqrt{9-x}}$$

$$f''(x) = \frac{3(x-12)}{4(9-x)^{3/2}} = 0 \text{ when } x = 12.$$

$x = 12$ is not in the domain. f'' is not continuous at $x = 9$.

Interval:	$-\infty < x < 9$
Sign of f'':	$f'' < 0$
Conclusion:	Concave downward

Concave downward: $(-\infty, 9)$

No point of inflection

23. $f(x) = \dfrac{4}{x^2 + 1}$

$f'(x) = \dfrac{-8x}{\left(x^2 + 1\right)^2}$

$f''(x) = \dfrac{8\left(3x^2 - 1\right)}{\left(x^2 + 1\right)^3}$

$f''(x) = 0$ for $x = \pm\dfrac{\sqrt{3}}{3}$

Intervals:	$-\infty < x < -\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3} < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $\left(-\infty, -\dfrac{\sqrt{3}}{3}\right), \left(\dfrac{\sqrt{3}}{3}, \infty\right)$

Concave downward: $\left(-\dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3}\right)$

Points of inflection: $\left(-\dfrac{\sqrt{3}}{3}, 3\right)$ and $\left(\dfrac{\sqrt{3}}{3}, 3\right)$

24. $f(x) = \dfrac{x + 3}{\sqrt{x}}$, Domain: $x > 0$

$f'(x) = \dfrac{x - 3}{2x^{3/2}}$

$f''(x) = \dfrac{9 - x}{4x^{5/2}} = 0$ when $x = 9$

Intervals:	$0 < x < 9$	$9 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$
Conclusion:	Concave upward	Concave downward

Concave upward: $(0, 9)$

Concave downward: $(9, \infty)$

Points of inflection: $(9, 4)$