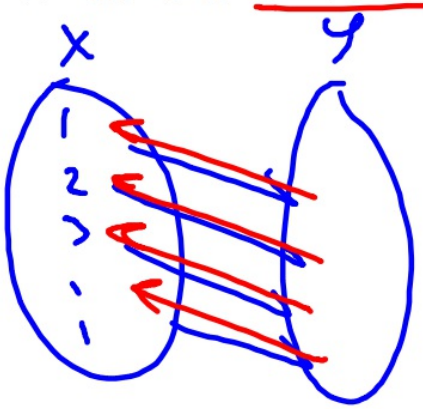
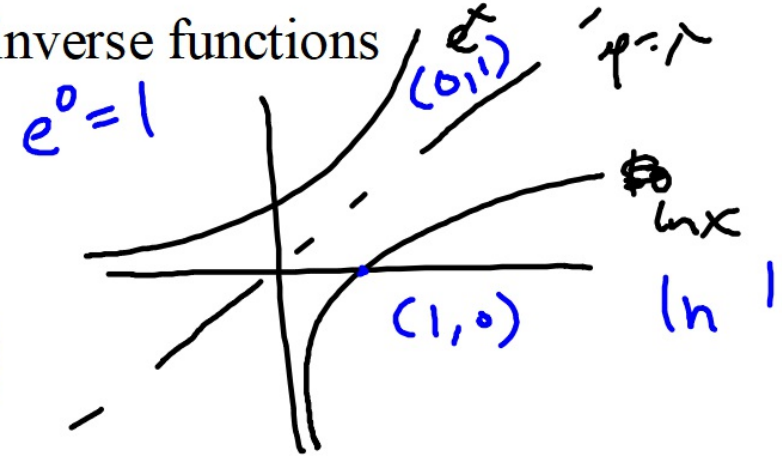
Calculus DS Mini Lesson: derivatives of inverse functions

What is an inverse function?



$f:(a,b)$

$f^{-1}:(b,a)$



Find the inverse function for $y=5x+2$ (4, 22)

replace x with y and v.v.

$$x = 5y + 2$$

$$x - 2 = 5y$$

$$\frac{1}{5}(x-2) = y$$ (22, 4)

A fundamental property of inverse functions:

If f and g are inverse functions, then:

$$f(g(x)) = g(f(x)) = x$$

ex: $f: x^2$, $g: \sqrt{x}$

$f(g(x))$

$f(\sqrt{x})$

$(\sqrt{x})^2 = x$

Inverse Deriv. Formula: f & g are inverses.

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

Let $f(x)=x^5+2x-1$ and let g be the inverse of f. Find g'(2)

1. Set original function equal to inverse's input.
2. Take derivative of original fct
3. Use the inverse deriv. formula

① $f(x) = 2$

$x^5 + 2x - 1 = 2$

guess/check:

$x=1$

$1^5 + 2(1) - 1 = 2$

$2 = 2$ ✓

$f:(1,2)$

$g:(2,1)$ by inverses

② $f'(x) = 5x^4 + 2$

③ $g'(x) = \frac{1}{f'(g(x))}$

$g'(2) = \frac{1}{f'(g(2))}$

$g'(2) = \frac{1}{f'(1)} = \frac{1}{5(1)^4+2} = \frac{1}{7}$

Let $f(x)=5-2x^3$ and let g be the inverse of f. Find g'(7)

Arabic Numbers		
٠	0	صفر
١	1	واحد
٢	2	اثنان
٣	3	ثلاثة
٤	4	أربعة
٥	5	خمسة
٦	6	ستة
٧	7	سبعة
٨	8	ثمانية
٩	9	تسعة
١٠	10	عشرة

$f(x) = 7$

$5-2x^3 = 7$

$-2 = 2x^3$

$-1 = x^3$

$\sqrt[3]{-1} = x$

$-1 = x$

$f:(-1, 7)$

$g:(7, -1)$

1. Set original function equal to inverse's input.
2. Take derivative of original fct
3. Use the inverse deriv. formula

$f'(x) = -6x^2$

$g'(x) = \frac{1}{f'(g(x))}$ $\quad g'(7) = \frac{1}{f'(g(7))} = \frac{1}{f'(-1)} = \frac{1}{-6(-1)^2} = -1/6$

$g(7) = -1$

Let g be the inverse of f. Find g'(3) for $f(x)=\sqrt{x^2-3x-1}$ (domain: $x>4$)