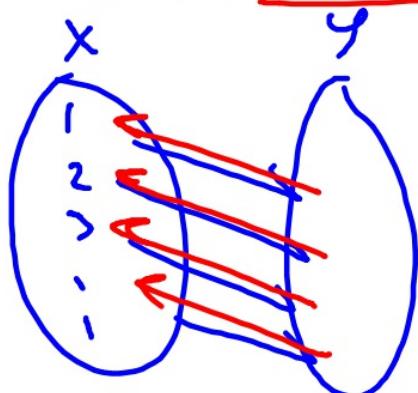
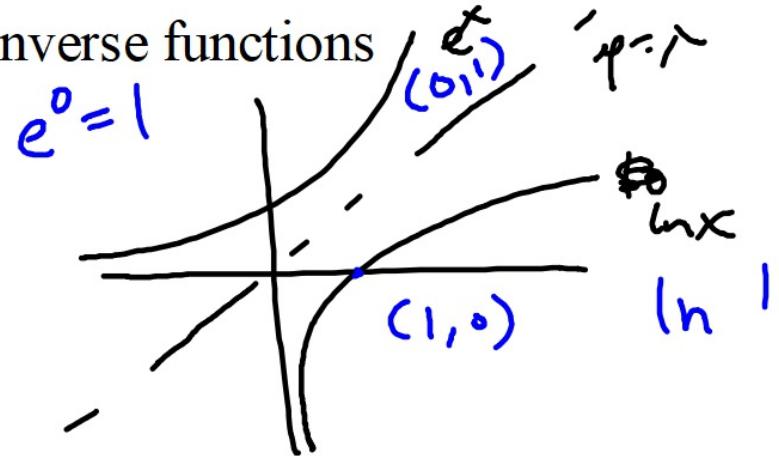


## Calculus DS Mini Lesson: derivatives of inverse functions

What is an inverse function?



$$f: (a, b) \rightarrow (b, a)$$



Find the inverse function for  $y=5x+2$  (4, 22)  
replace x with y  
and u·u·

$$x = 5y + 2$$

$$x - 2 = 5y$$

$$\underline{\underline{\frac{1}{5}(x-2) = y}} \quad (22, 4)$$

A fundamental property of inverse functions:

If  $f$  and  $g$  are inverse functions, then:

$$\underline{f(g(x)) = g(f(x)) = x}$$

The diagram illustrates the composition of inverse functions  $f$  and  $g$ . At the top, two functions are defined:  $f: x^2$  (in pink) and  $g: \sqrt{x}$  (in blue). A green horizontal line connects the outputs of  $f$  to the inputs of  $g$ . Below this, the expression  $f(g(x))$  is shown in pink, with its output also connected by a green line to the input of the third function,  $f(\sqrt{x})$ , shown in pink. Finally, the expression  $(\sqrt{x})^2$  is shown in blue, with its output connected by a green line to the result  $x$ , also shown in blue.

Inverse Deriv.  
Formula :  $f \circ g$  are  
inverses.

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$f'(g(x)) \cdot g'(x) = 1$$

$$g'(x) = \frac{1}{f'(g(x))}$$

Let  $f(x) = \underline{x^5 + 2x - 1}$  and let g be the inverse of  $f$ . Find  $g'(2)$

①  $\underline{f(x)} = 2$

$$\cancel{x^5} + 2x - 1 = 2$$

guess/check:

$$x=1$$

$$1^5 + 2(1) - 1 = 2$$

②  $f'(x) = 5x^4 + 2$   $\overset{x=2}{\cancel{2=2}}$   $\frac{f:(1,2)}{g:(2,1)}$  by inverses

③  $g'(x) = \frac{1}{f'(g(x))}$

$$g'(2) = \frac{1}{f'(g(2))}$$

$$g'(2) = \frac{1}{f'(1)} = \frac{1}{5(1)^4 + 2} = \frac{1}{7}$$

1. Set original function equal to inverse's input.
2. Take derivative of original fct
3. Use the inverse deriv. formula

Let  $f(x) = 5 - 2x^3$  and let  $g$  be the inverse of  $f$ . Find  $g'(7)$

| Arabic Numbers |          |
|----------------|----------|
| ٠              | ٠ صفر    |
| ١              | ١ واحد   |
| ٢              | ٢ اثنان  |
| ٣              | ٣ ثلاثة  |
| ٤              | ٤ أربعة  |
| ٥              | ٥ خمسة   |
| ٦              | ٦ ستة    |
| ٧              | ٧ سبعة   |
| ٨              | ٨ ثمانية |
| ٩              | ٩ تسعة   |
| ١٠             | ١٠ عشرة  |

$$f(x) = 7$$

$$5 - 2x^3 = 7$$

$$\begin{aligned} -2 &= 2x^3 \\ -1 &= x^3 \\ \sqrt[3]{-1} &= x \end{aligned}$$

$$f'(x) = -6x^2$$

1. Set original function equal to inverse's input.
2. Take derivative of original fct
3. Use the inverse deriv. formula

$$g'(x) = \frac{1}{f'(g(x))} = g'(7) = \frac{1}{f'(\cancel{g(7)})} = \frac{1}{f'(-1)} = \frac{1}{-6(-1)^2} = \frac{1}{-6(1)} = \frac{1}{-6} = -\frac{1}{6}$$

Let  $g$  be the inverse of  $f$ . Find  $g'(3)$  for  $f(x) = \sqrt{x^2 - 3x - 1}$  (domain:  $x > 4$ )