Assessment on Related Rates

When finished, you may look at curve sketching hw sols located by basket.
Be sure to put them back when you are done with them.

Linearization: AD-14

Important concept: Local Linearity

In your calculator, graph Y1 = $\sin^{-1}(x)$

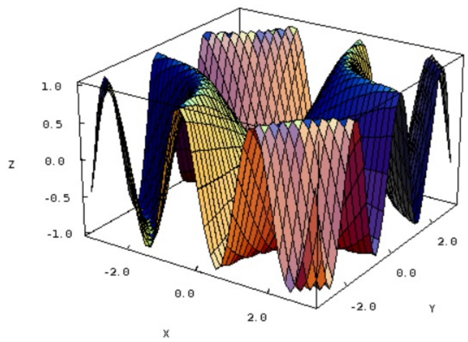
use the TRACE tool to highlight any non-extrema point you want
then use ZOOM a bunch of times

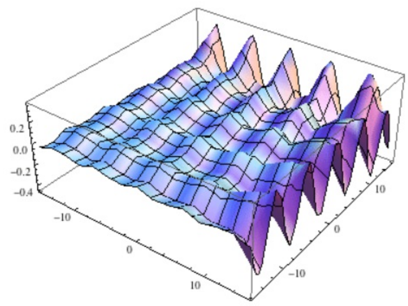
Local Linearity:
All functions behave linearly infinitely close to a particular point.

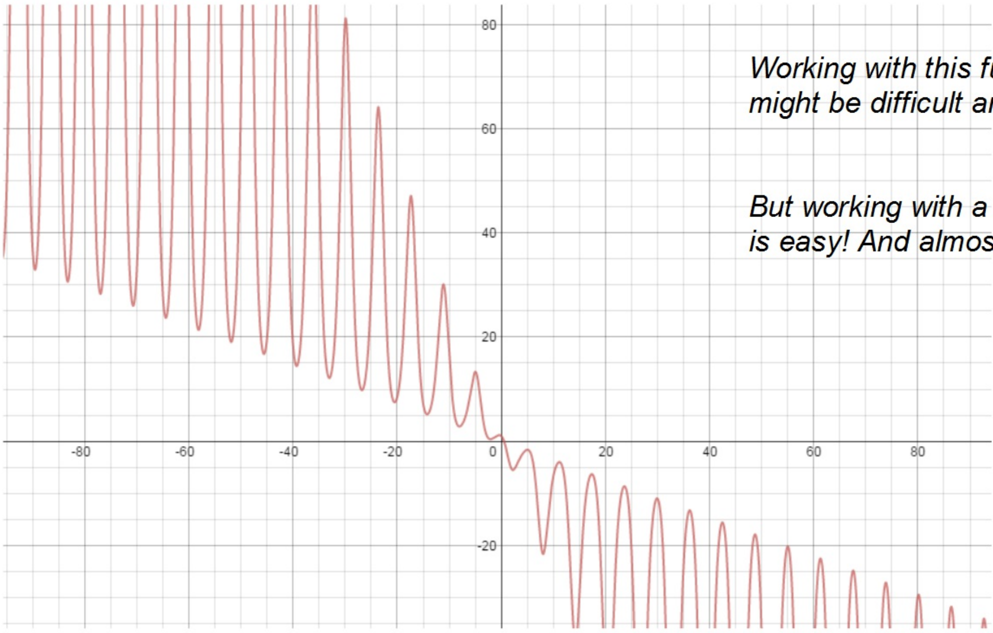
Meaning: the line tangent to a function f(x) at x = c (for some value c) serves as a good approximation for the function at values "near" c

So what?

Engineers, scientists, statisticians: all use processes like these to take compliated, cumbersome, difficult behavior and instead use approximation methods to make them easier to calculate with only very small (and calculable) error.





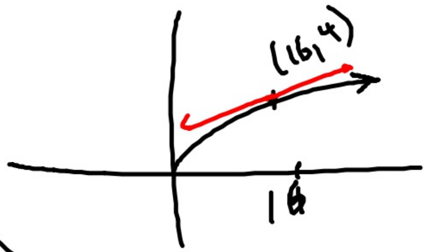


Working with this function might be difficult and time-consuming

But working with a simple y=mx+b line is easy! And almost the same thing.

How to use it!

Estimate the value of $\sqrt{16.5}$



$y = \sqrt{x} = x^{1/2}$

Tangent line @ (16,4)

$$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

$$y'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{2\cdot 4} = \frac{1}{8}$$

$$y - 4 = \frac{1}{8}(x - 16)$$

tan line equation

plug $16.5 = x$.

$$y - 4 = \frac{1}{8}(16.5 - 16)$$

$$y = \frac{1}{16} + 4$$

$$4\frac{1}{16}$$

AP time!

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

$(4 + \sin x)^{1/2}$

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

$y' = \frac{1}{2}(4 + \sin x)^{-1/2} \cdot (\cos(x))$

$y' = \frac{1}{2(4 + \sin(x))^{1/2}} \cdot \cos(x)$

$y'(0) = \frac{1}{4}$ $\quad \frac{1}{4} \cdot 1$

$y(0) = 2 \quad (0, 2)$

$y - 2 = \frac{1}{4}(x - 0)$

$y - 2 = \frac{1}{4}(.12)$

$y = 2.03$

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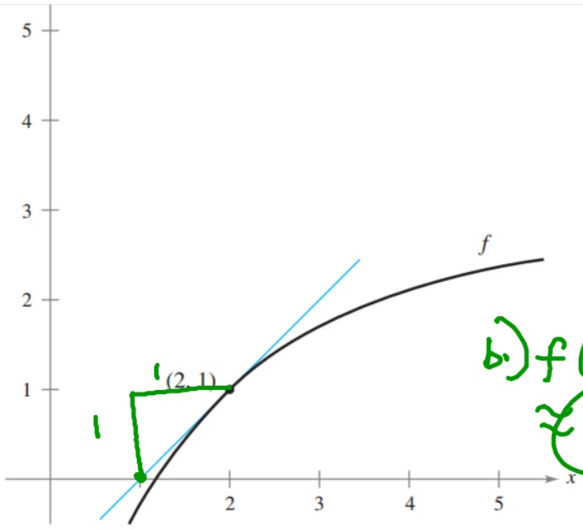


Estimate f(1.9) and f(2.1)

$m = 1 \quad (2,1)$

$y - 1 = 1(x - 2)$

b.) $f(2.1) \approx 1.1$

a.) $f(1.9) \approx$

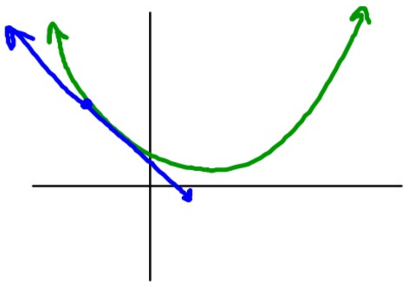
$y - 1 = 1(1.9 - 2)$

$y - 1 = -0.1$

$y = 0.9$

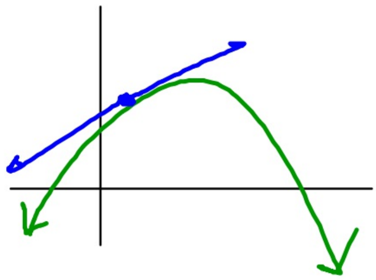
How does concavity impact approximations?

Sketch a concave up curve and a line tangent to it....



linear approx is an underestimate

Sketch a concave down curve and a line tangent to it....



linear approx is overestimate

#1-6; #21

(ignore book instructions; instead, use the functions and the given point to estimate f(2.2))