Good afternoon: no warm up, we'll randomize, get our tests back, then continue analyzing functions with derivatives

Note:
Last Q2 test: 12/13
Retake/Upgrade/Enrichment Day: 12/18
Q2 ends in <2 weeks!

visibly random grouping

Assessments

Fixing some common errors:

- don't use the word IT
- plug into F for absolute extrema (output)
  plug into F' for relative extrema (inc/dec)
- Why was it an underapproximation?
- Units for derivatives will always be a rate (unit1 per unit2)

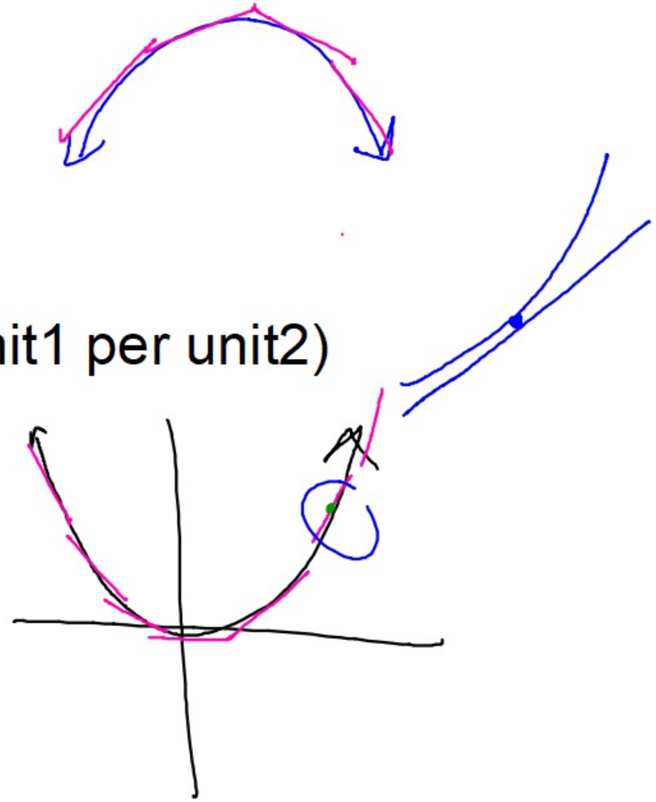
$y = x^2$

$y' = 2x$

$y'(3) = 6$

$(3, 9)$

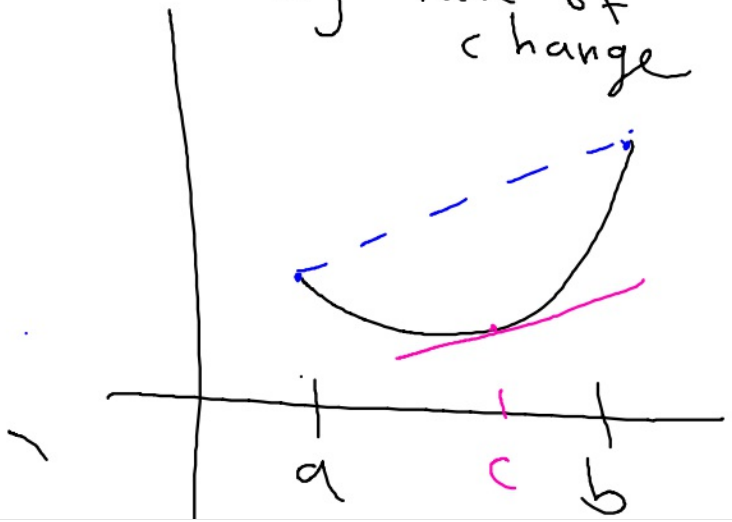
$y - 9 = 6(x - 3)$



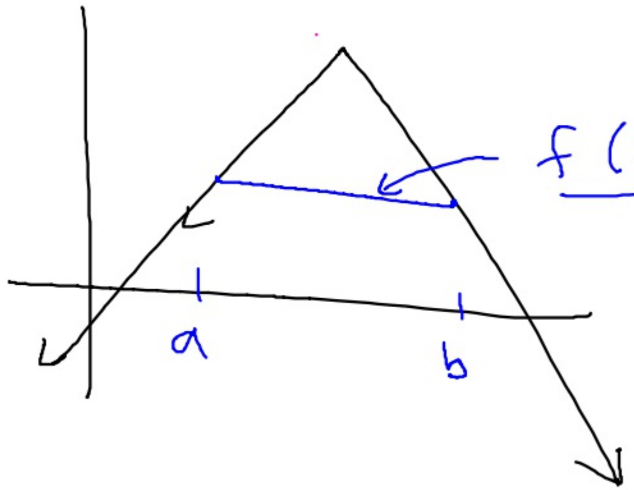
Mean Value Theorem

for a diff. function

$$\underbrace{\frac{f(b)-f(a)}{b-a}}_{\text{avg. rate of change}} = \underbrace{f'(c)}_{\text{instantaneous rate of change}} \quad \text{for some } c \in (a,b).$$



Why is continuity an insufficient requirement?



$$\frac{f(b)-f(a)}{b-a} = m$$

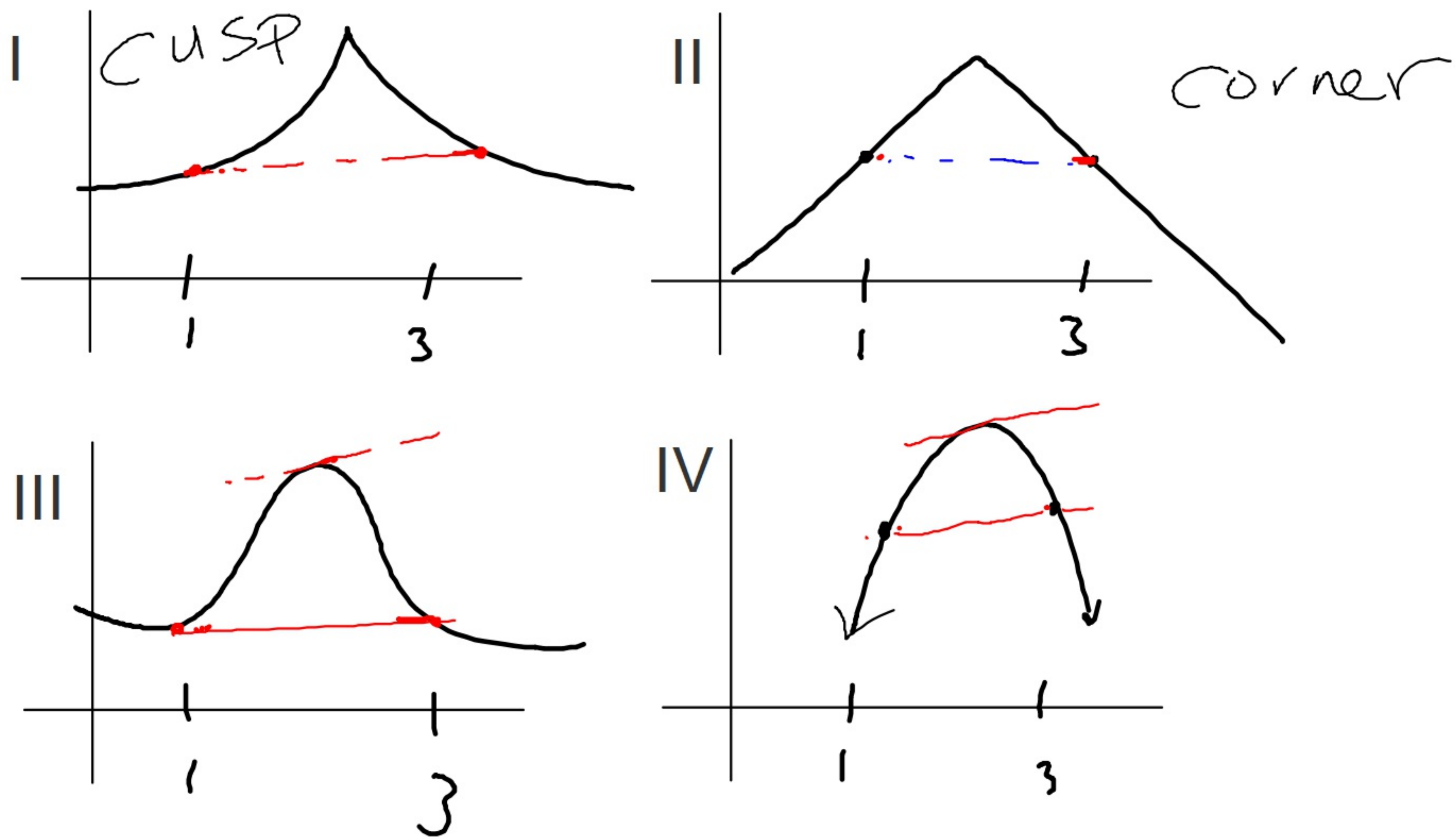
Why is differentiability a requirement?

Continuity is a requirement for the IVT

MVT

Differentiability is a requirement for the ~~IVT~~

Since F's differentiability implies the continuity of F', the MVT is sort of like using the IVT for F'



All 4 are continuous, but only III and IV are differentiable
see how MVT fails for I and II?

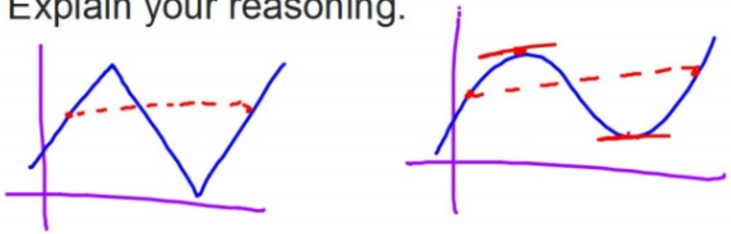
Have we seen this before??

2nd warm up :)

Your car enters a toll highway at 1pm. The highway stretches for 120 miles and has a speed limit of 55mph. You come to the toll booth at the end of the highway at 3pm and are handed a speeding ticket.

Why? Explain your reasoning.

Mean Value Theorem

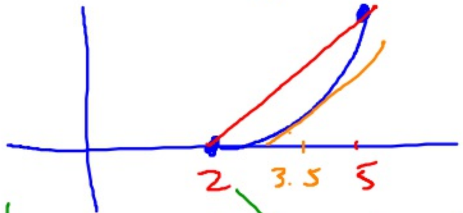


avg.

$$\frac{120}{2} = 60\text{mph}$$

August 2018

Find the value(s) of c guaranteed to exist by the MVT for $f(x)=x^2-6x+8$ on [2,5]



$f'(x) = 2x-6$

$\longrightarrow$ f is diff (f cont, f' cont) $\longrightarrow$ MVT applies

① Find the avg. rate of change over interval:

$(2, 0)$ $\qquad$ $(5, 3)$ $\qquad$ $\frac{3}{3} = 1$

$4-6(2)+8$ $\qquad$ $25-30+8$

② Set f' = avg. rate $\longrightarrow$ $2x-6=1$

$\underline{x = 3.5}$

Find c MVT.

$f(x) = x^2 - 4x + 9 \quad [-3, 1]$

$f(1) = 6$

$f(-3) = 30$

$\frac{6-30}{1--3} = \frac{-24}{4} = -6$

MVT

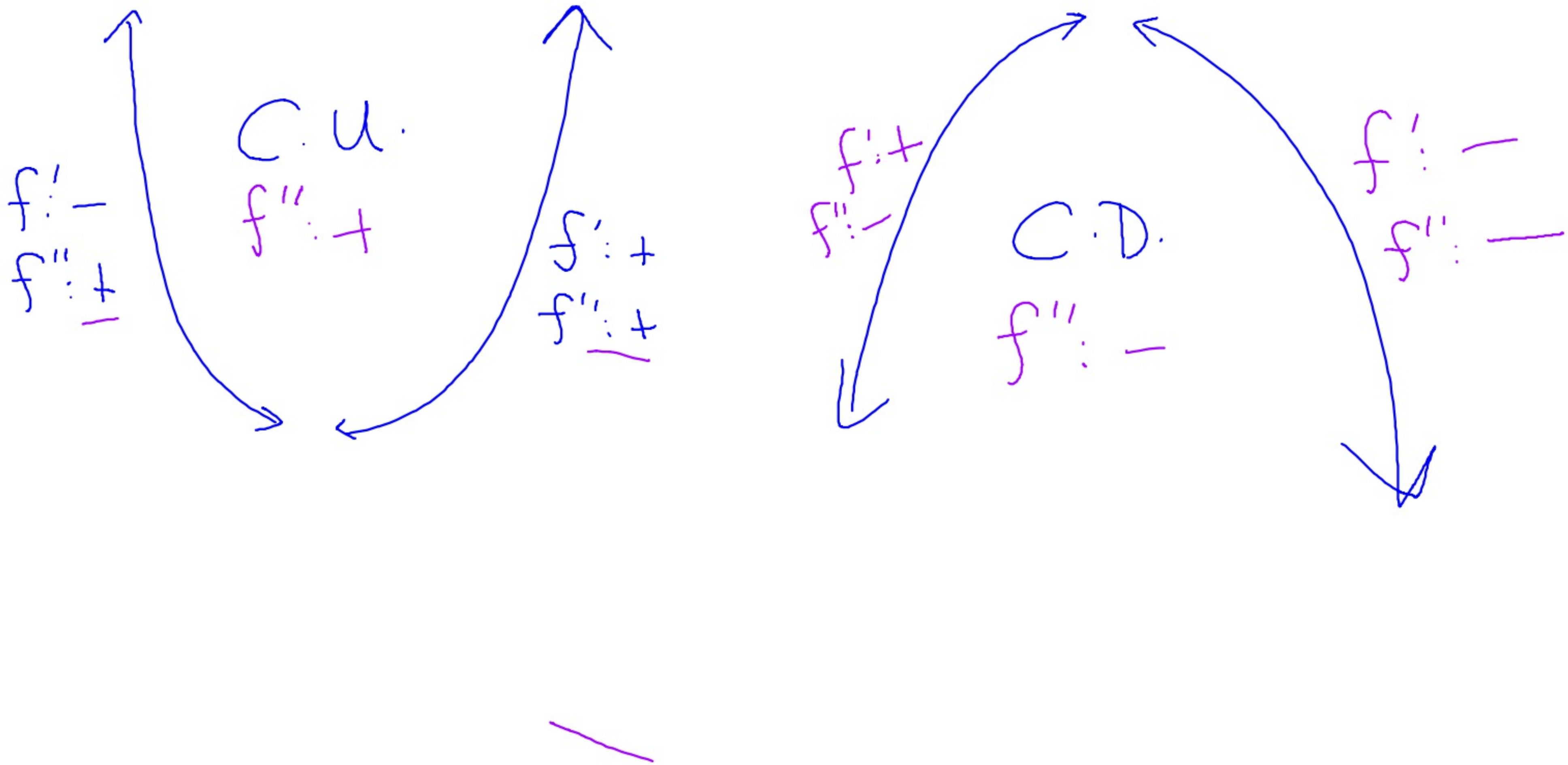
$f'(c) = 2c - 4 = -6$

$2x = -2$

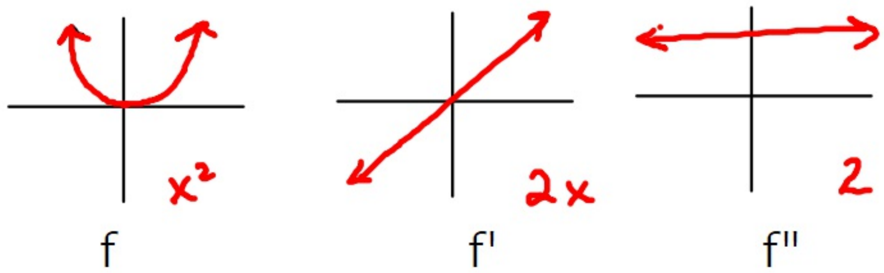
$x = -1$

Share with your face partner something you've learned so far today

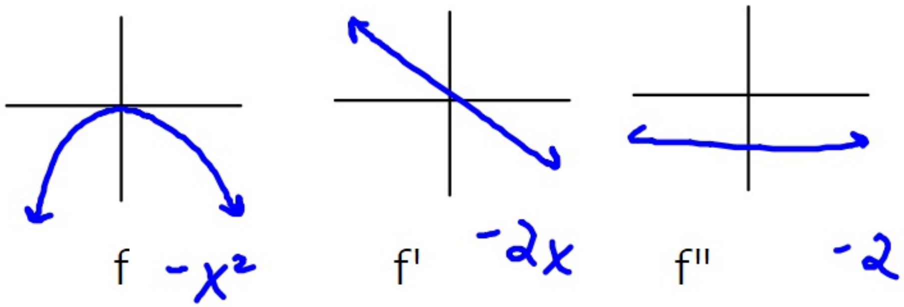
Review: the 4 kinds of curvature



Concave up

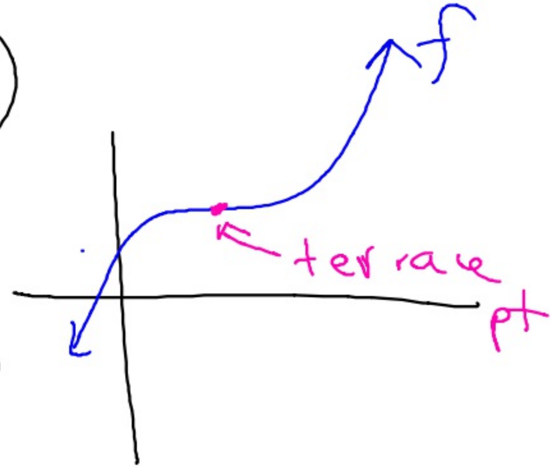


Concave down



Terrace Points

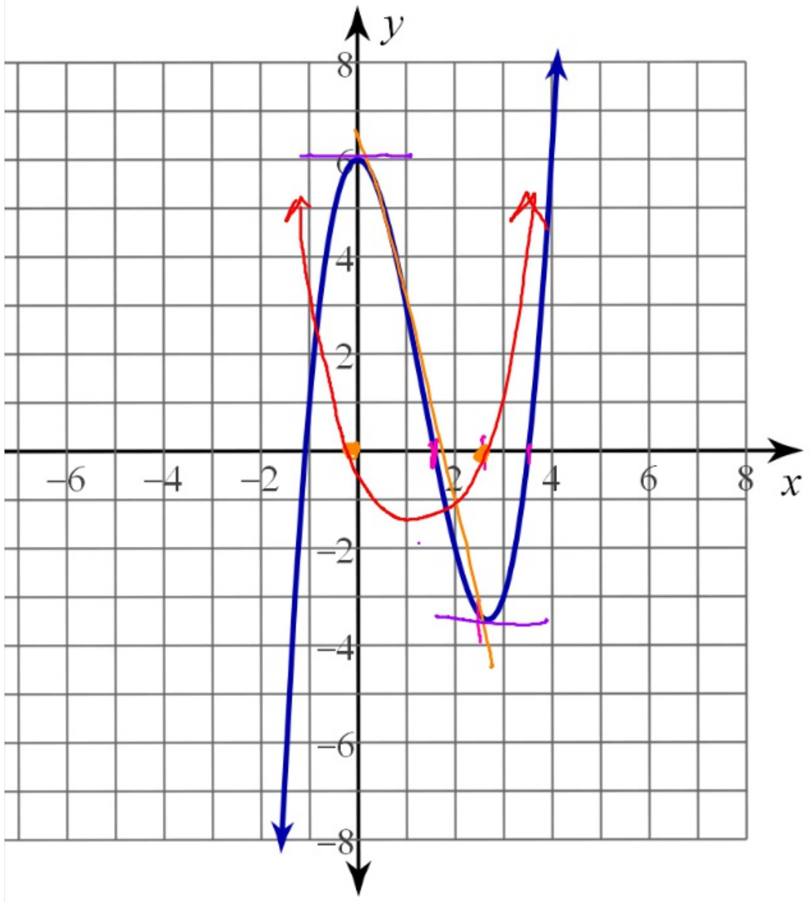
when $f''(x) = 0$ or undefined.
(like C.N. but for f'')



Inflection Points (change in concavity)

occur at terrace pts.
→ must have a sign change

Here's F'



Where is F increasing?

$(f'\ \text{pos})$ $(-1, 1.5)$ $(3.5, \infty)$

Where does F have a relative min?

$x = -1,\ x = 3.5$

Where is F concave up? $(-\infty, 0)\ (2.5, \infty)$

$f''\ \text{pos} \rightarrow f'\ \text{inc.}$

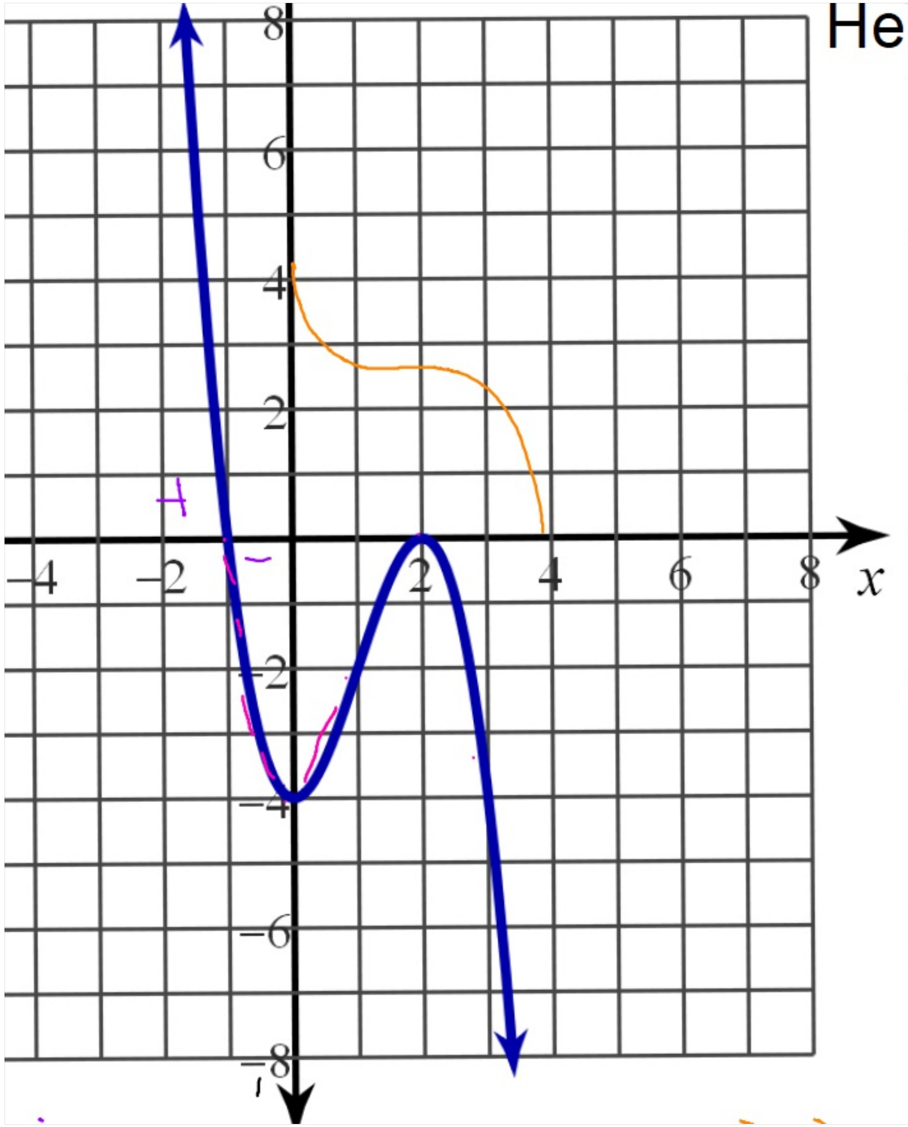
Where is F concave down?

$f''\ \text{neg} \rightarrow f'\ \text{dec}$ $(0, 2.5)$

Where is/are the inflection points for F?

$(f'' = 0;\ \text{sign change})$

$x = 0,\ x = 2.5$



Here's F'

Where is F decreasing?

(-1, ∞) (-1, 2) (2, ∞)

Where does F have a relative max?

x = -1

Where is F concave up?

(0, 2) f' inc

Where is F concave down?

(-∞, 0) (2, ∞) (f' dec)

Where is/are the inflection points for F?

x = 0, x = 2

b/c f'' changes signs

What's funky about x=2?

CN but not extreme

is an I.P.

Find/justify the location of all inflection points for $f(x) = x^3 - x^2 - 1$

$$f'(x) = 3x^2 - 2x$$

$$f''(x) = 6x - 2 = 0$$

- change in concavity
- second deriv.

$$\underbrace{x = \frac{1}{3}}_{\text{terrace pt}}$$

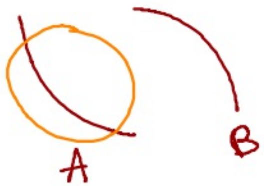
f''	$-$		$+$
x		$\frac{1}{3}$	
f'	dec		inc
f	C.D		C.U.

f has an I.P @ $x = 1/3$ b/c sign change in f''

Putting it all together:

Describe the nature of the curvature at x=1/4 for f(x)= $-x^3+2x^2-x$

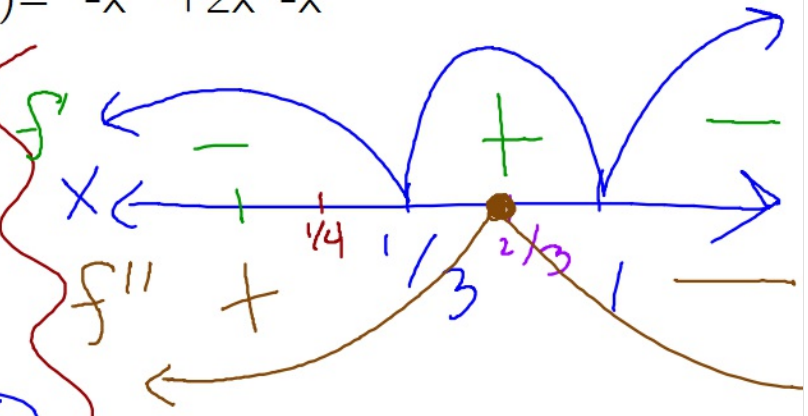
$$f' = -3x^2 + 4x - 1 = 0$$



$$-1(3x^2 - 4x + 1) = 0$$

$$-1(3x-1)(x-1) = 0$$

$$\underbrace{x = 1/3 \qquad x = 1}_{C.N.}$$



$$f'' = -6x + 4 = 0$$

$$6x = 4$$

$$\underbrace{x = 2/3}_{T.P.}$$

$x = 1/4$

f is dec,

Concave up.

Give any intervals where the function is decreasing and concave up

$f(x)= x^3-2x^2$

HW

p. 175 #37-42

p 192 #15-22, 53-54