Good afternoon: no warm up, check the hw answers to extrema AP multiple choice q's

01 C	07 B
02 B	08 D
03 C	09 C
04 A	10 E
05A	11 D
06 D	

First Q3 test:
Tuesday

Visibly Random Grouping

Continuing related rates

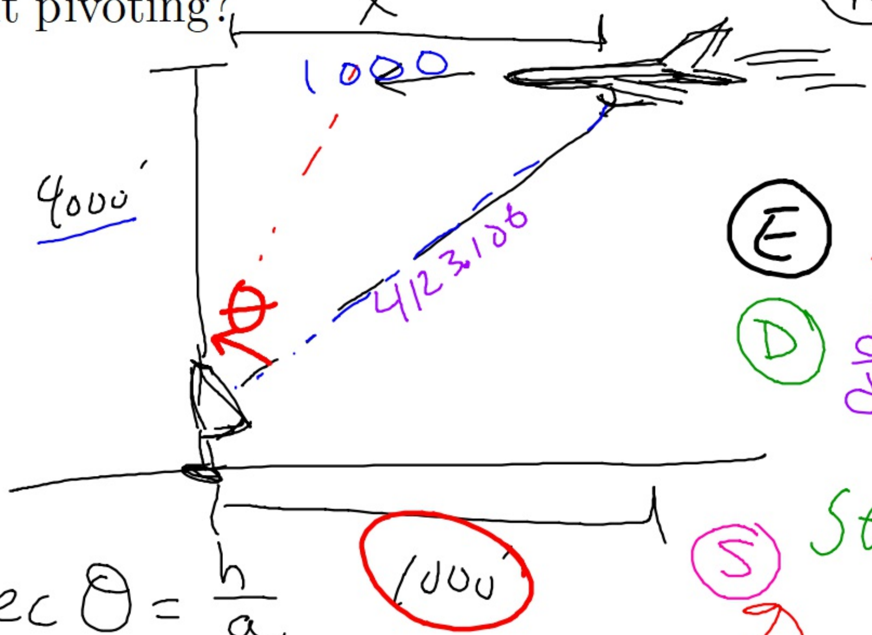
A plane is flying due west overhead at an altitude of 4000ft at a speed of 700 ft/s. A searchlight on the ground is angled up and tracking the plane. When the plane is 1000 feet from the light, how quickly is the light pivoting?

N

W E

R: $\frac{dx}{dt} = -700$

D



E: $x = 1000$

$$\tan\theta = \frac{x}{4000}$$

D: $$\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{x}{4000}$$

S: $$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{4000}\cdot\frac{dx}{dt}$$

$$(1.031)^2 \cdot \frac{d\theta}{dt} = \frac{1}{4000}\cdot -700$$

$$1.0625 \cdot \frac{d\theta}{dt} = -0.175$$

$$\boxed{\frac{d\theta}{dt} = -0.165}$$

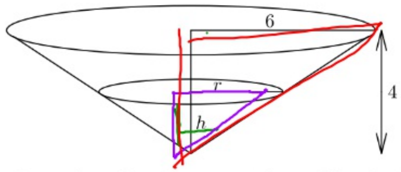
$$\sec\theta = \frac{h}{a}$$

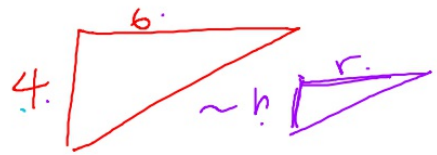
$$\cos\theta = \frac{a}{h} = \frac{4000}{4123.106}$$

$$\cos\theta = .970$$

$$\sec\theta = \frac{1}{.970} = 1.031$$

D





7. The conical reservoir shown above has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of

A) 0.177 ft per min
B) 0.354 ft per min
C) 0.531 ft per min
D) 0.708 ft per min
E) 0.885 ft per min

R

$D = 12 \to r = 6$

4

$h = 2$

$\frac{dV}{dt} = 10$

$\frac{dh}{dt} = ?$

$\frac{6}{r} \Rightarrow \frac{4}{h}$

$6h = 4r$

$\frac{6h}{4} = r$

$\frac{3}{2}h = r$

E

$V = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{1}{3}\pi\left(\frac{3}{2}h\right)^2 \cdot h$

D

$\frac{dV}{dt} =$

D

$V = \frac{1}{\cancel{3}}\pi \cdot \frac{\cancel{9}\,3}{4}h^3 = \frac{3\pi}{4}h^3$

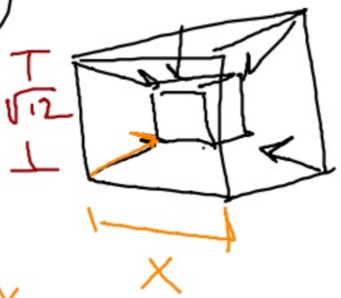
$\frac{dV}{dt} = \frac{3\pi}{4} \cdot 3h^2 \cdot \frac{dh}{dt}$

S

$\frac{10}{9\pi} = \frac{dh}{dt} \leftarrow 10 = \frac{9\pi}{4} \cdot \cancel{(2)^2} \cdot \frac{dh}{dt}$

An ice cube is melting. The rate at which the sides are decreasing is 0.5 inches per hour. Find the rate at which the volume is decreasing when the surface area of the cube is 72 square inches.

D



$\frac{dx}{dt} = -0.5$

E $V = x^3$

R $\frac{dx}{dt} = -0.5$

$SA = 72 = 6x^2$

$12 = x^2$

$\sqrt{12} = x$

D $\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

? ✓ ✓

S $\frac{dV}{dt} = 3 \cdot (\sqrt{12})^2 \cdot -0.5)$

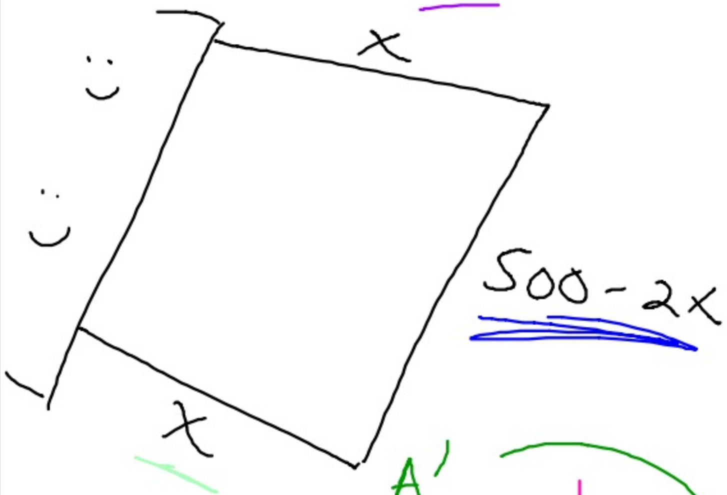
$\boxed{\frac{dV}{dt} = -18 \text{ in}^3/\text{hr}}$

Optimization

Applying calculus to find maximum or minimum of some descriptive function subject to constraints

Our rectangular yard needs a fence. We have 500 feet of fencing material and a building is on one side of the yard and so won't need any fencing. What should the dimensions of the fenced yard be to have the maximum area?

optimize



$$A = x(500-2x)$$

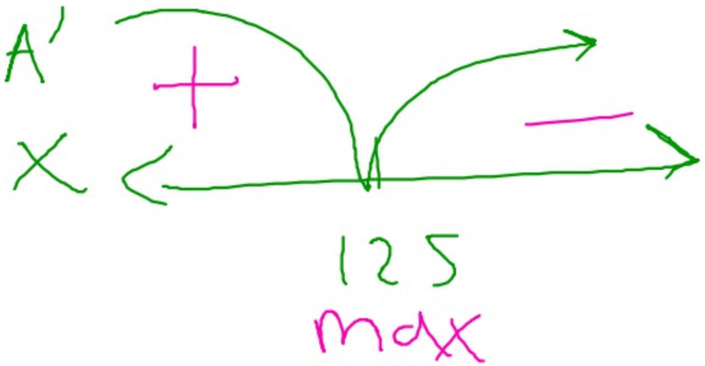
$$A = 500x - 2x^2$$

$$\frac{dA}{dt} = 500 - 4x = 0$$

$$500 = 4x$$

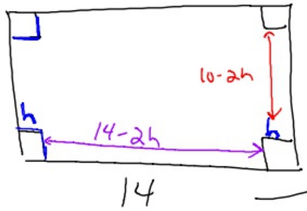
$$125 = x$$

C.N.



125 x 250

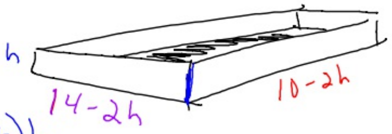
You have a piece of cardboard that is 14 inches by 10 inches and you're going to cut out the corners and fold up the sides to form a box. Determine the height of the open box that will give a maximum volume.



opt.

$V = (14-2h)(10-2h)h$

$2(7-h)2(5-h)h$



$V = 4h(35-12h+h^2)$

$4(35h-12h^2+h^3)$

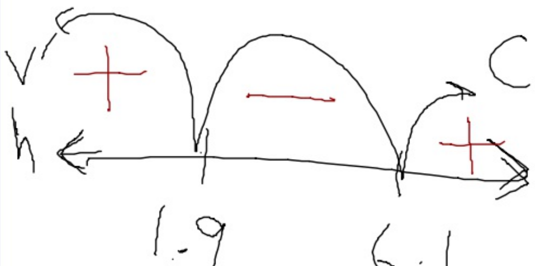
$V = 140h - 48h^2 + 4h^3$

$\frac{dV}{dh} = 140 - 96h + 12h^2 = 0$

$\rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = 6.082, 1.918$

C.N



Find two numbers whose sum is 40 and whose product is as large as possible.

not done in class

Constraint: $x + y = 40 \rightarrow y = 40 - x$

optimize:

$$P = x \cdot y$$

$$P = x(40 - x)$$

$$P = 40x - x^2$$

$$P' = 40 - 2x = 0$$

$$x = 20$$

CN

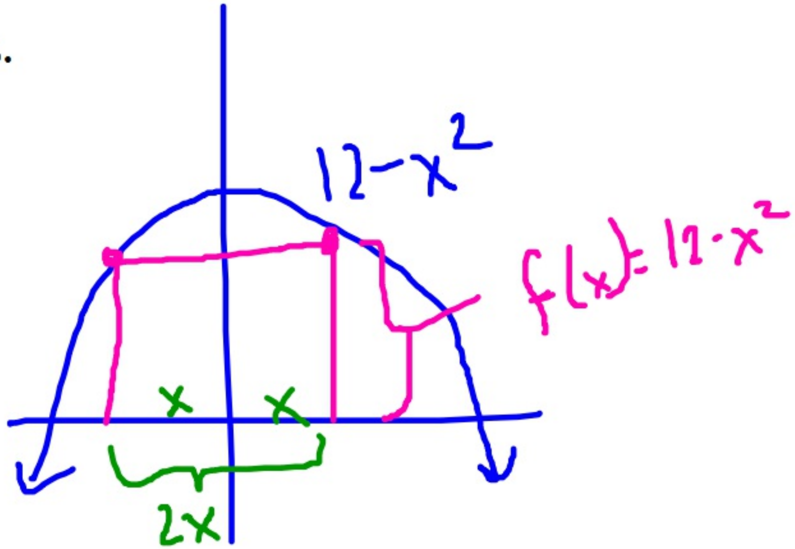
p' + | −

x 20

MAX

both 20

A rectangle whose base is on the x-axis has two vertices which lie on the parabola 12-x^2. Find the dimensions and area of the rectangle with largest area.



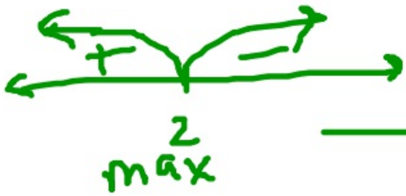
not done in class

$$A = 2x(12-x^2)$$

$$A = 24x - 2x^3$$

$$A' = 24 - 6x^2 = 0$$

$$x = 2$$



if x=2, then f(2)=12-4=8

so 8 is the height

length: 4, height 8; A=32

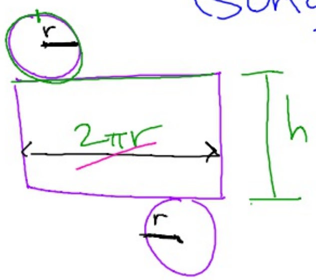
You have 108 square inches of material to make a square base box. Find the dimensions to maximize the box volume.

Find the dimensions of a 1-liter (1000ml = 1000cc) cylindrical can that will use the least material.

Constraints: $V = \pi r^2 \cdot h = 1000$

$h = \frac{1000}{\pi r^2}$

(Surface Area)



optimize:

$$SA = 2 \cdot \pi r^2 + 2\pi r h$$

$$SA = 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \frac{1000}{\cancel{\pi r^2}}$$

$$SA = 2\pi r^2 + \frac{2000}{r}$$

$+2000r^{-1}$

$$SA' = 4\pi r - 2000r^{-2}$$

factor out lesser exponent

$$SA' = r^{-2}(4\pi r^3 - 2000)$$

$$SA' = \frac{4\pi r^3 - 2000}{r^2}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \rightarrow 5.419 \text{ cm}$$

$$h = \frac{1000}{\pi r^2}$$

$$10.839 \text{ cm}$$

note that the height
is equivalent to the diameter
aka a 'square' cylinder is best

HW

related rates: p 153 #15-18, 21

optimization: p. 220 #17-20, 25