Good afternoon: warm ups

If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

(A) $\dfrac{\sin(\ln(2x))}{2x}$ (C) $\dfrac{\cos(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$ (D) $\cos\left(\dfrac{1}{2x}\right)$

$\cos(\ln(2x)) \cdot \dfrac{1}{\cancel{2}x} \cdot \cancel{2}$

The local linear approximation to the function g at $x = \frac{1}{2}$ is $y = 4x + 1$. What is the value of $g\left(\frac{1}{2}\right) + g'\left(\frac{1}{2}\right)$?

$y' = 4$

(A) 4 (C) 6

(B) 5 (D) 7

$3 + 4$

The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

(A) The cost to shred 500 pounds of documents is \$80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

(D) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.

$$\lim_{x\to 0} \frac{7x - \sin x}{x^2 + \sin(3x)} = \frac{0}{0}$$

(A) 6 (C) 1

(B) 2 (D) 0

$$\frac{7 - \cos(x)}{2x + 3\cos 3x} \to \frac{7-1}{0+3}$$

$6/3$

Visibly Random Grouping

This week: finishing some assorted derivative topics

Today: Related Rates

Weds DS: Inverses

Thurs: More RR, Optimization

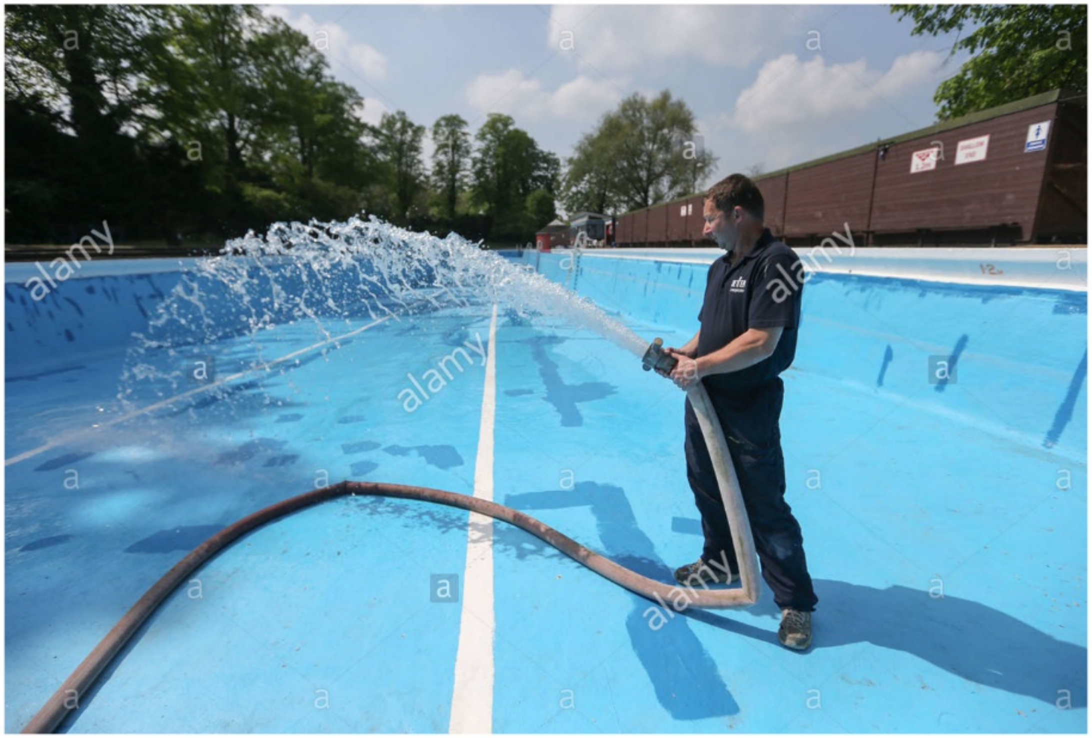
Next week: starting antiderivatives/integration

$$\int$$

Blowing up a balloon

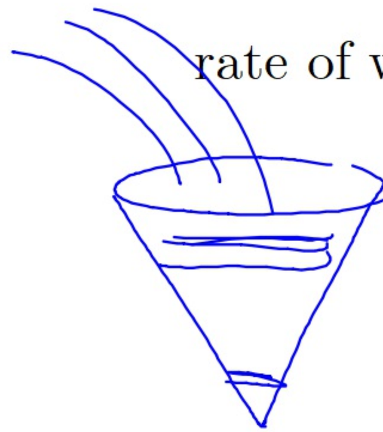
constant volume

What else changes?
And how?



rate of water coming
out of hose

rate of water depth



Understanding "derivative with respect to"

Suppose x and y are functions of time t. Find dy/dt for

$$x(t) \qquad y(t)$$

$$\frac{d}{dt}\left(3x^2+5y^2\right)=\frac{d}{dt}\left(12\right)$$

$$6x\cdot\frac{dx}{dt}+10y\cdot\boxed{\frac{dy}{dt}}=0$$

$$\frac{\cancel{10y}\cdot\frac{dy}{dt}}{\cancel{10y}}=\frac{-6x\cdot\frac{dx}{dt}}{10y}$$

$$\frac{dy}{dt}=\frac{-6x\cdot\frac{dx}{dt}}{10y}$$

Implicit differentiation, doing d/dt to both sides instead of d/dx

D Diagram

R Rates (label given information, needed information)

E Equation (geometry formula, usually. Be sure needed info is only unknown)

D Differentiate (with respect to TIME)

S Substitute, then solve

The Falling Ladder Problem

http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_app_rr_falling_ladder.html

gsp

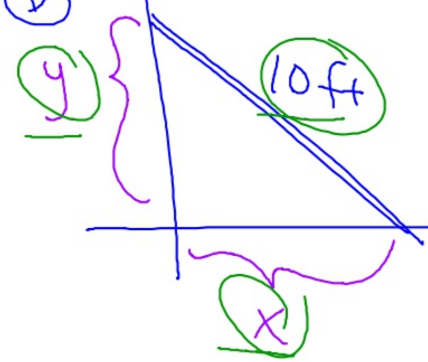
A 10 foot ladder is leaning against a wall. It has rained and turned the ground into a muddy, slippery mess. A gust of wind of constant velocity nudges the ladder so that the base of the ladder slides away from the wall at a rate of 4 feet per second.

How fast is the top of the ladder sliding down the wall when the base of the ladder is 8 feet from the wall?

R $\frac{dy}{dt} = ?$ $\frac{dx}{dt} = 4$

$x = 8$

D



E $\frac{d}{dt}(x^2 + y^2) = (10^2)\frac{d}{dt}$

D $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

Need y

$8^2 + y^2 = 10^2$

$y = 6$

S

$\frac{dy}{dt}$ $\frac{ft}{sec}$

$2(8)\cdot(4) + 2(6)\left(\frac{dy}{dt}\right) = 0$

$64 + 12\frac{dy}{dt} = 0$

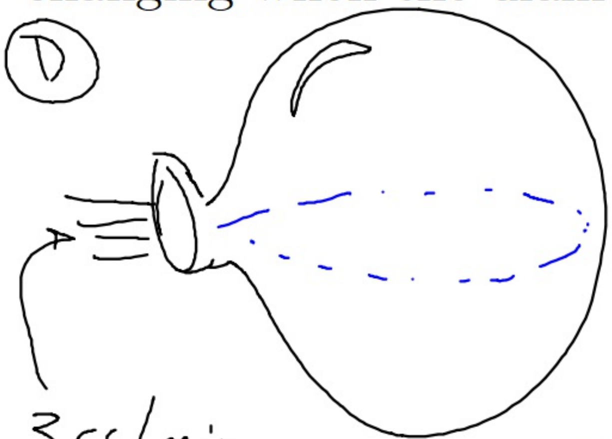
$12\frac{dy}{dt} = -64$

$\frac{dy}{dt} = -64/12 \rightsquigarrow -16/3$ ft/sec

A spherical balloon is being inflated by a pump at a rate of 3 cubic centimeters per minute. At what rate is the radius of the balloon changing when the diameter of the balloon is 4 cm?

$\leftarrow V$

D



3 cc/min

R $\frac{dV}{dt} = 3$

$D = 4$

$(R = 2)$

$$\frac{dr}{dt} = ?$$

E $$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)\frac{d}{dt}$$

D $$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$$

✓ ✓ ?

$$3 = 4\pi(\cancel{2})^2 \cdot \frac{dr}{dt}$$

4

$\leftarrow$

$$3 = 16\pi \cdot \frac{dr}{dt}$$

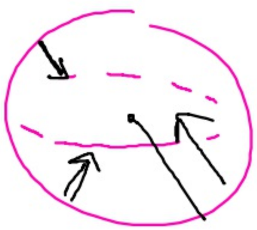
$$\boxed{\frac{3}{16\pi} = \frac{dr}{dt}}$$

cm/min

A spherical snowball melts such that its radius decreases at a rate of 2 in/min. At what rate is the volume of the snowball changing when the radius is 3 inches?

(wasn't in class)

D



-2 in/min

R $\frac{dV}{dt} = ?$ $\qquad \frac{dr}{dt} = -2$

getting smaller

$r = 3$

E $V = \frac{4}{3}\pi r^3$

D $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

?

✓ ✓

S $\frac{dV}{dt} = 4\pi \cdot 3^2 \cdot (-2)$

$\boxed{= -72\pi \ \text{in}^3/\text{min}}$

A ripple forms in a pond. It is expanding such that its radius grows by 2 inches per second. How fast is the area growing when the ripple's circumference is 32π in?

D

R $\frac{dr}{dt} = 2$

$C = 32\pi$

$(C = 2\pi r)$

$r = 16$

$\frac{dA}{dt} = ?$

E $A = \pi r^2$

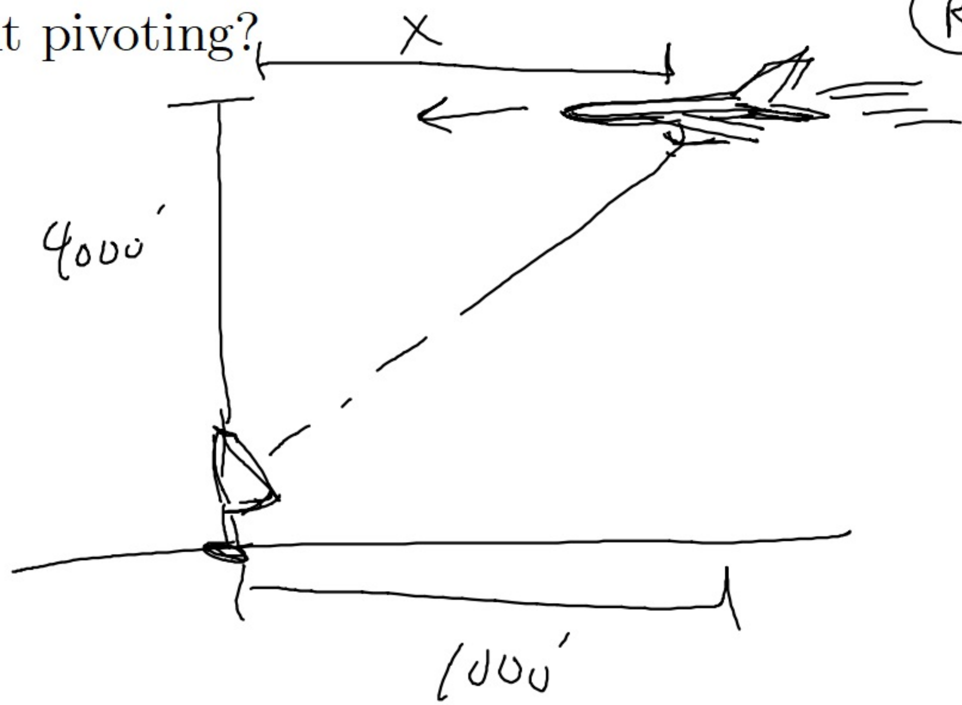
D $\frac{d}{dt} A = \frac{d}{dt} \pi r^2 \quad \pi \cdot 2r \frac{dr}{dt}$

$$\underbrace{\frac{dA}{dt}}_{?} = 2\pi \underbrace{r}_{16} \cdot \underbrace{\frac{dr}{dt}}_{2}$$

$$\frac{dA}{dt} = 2 \cdot \pi \cdot 16 \cdot 2 \rightarrow \boxed{64\pi \ \text{in}^2/\text{sec}}$$

A plane is flying due west overhead at an altitude of 4000ft at a speed of 700 ft/s. A searchlight on the ground is angled up and tracking the plane. When the plane is 1000 feet from the light, how quickly is the light pivoting?

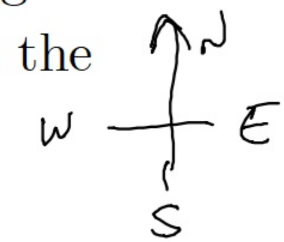
D



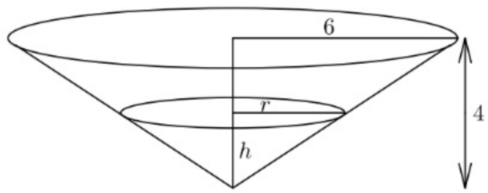
R

$$\frac{dx}{dt} = -700$$

$X = 1000$



TBD



7. The conical reservoir shown above has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of

A) 0.177 ft per min

B) 0.354 ft per min

C) 0.531 ft per min

D) 0.708 ft per min

E) 0.885 ft per min

- #1-11 on extrema handout from yesterday (due Thursday)

- p.153 (due Monday)

  #15-18, 21

  Don't forget calcchat