Good afternoon and welcome back! Please complete the warm-up in your notebooks

Find the equation of the line that is <u>normal</u> (perpendicular) to the curve $y=\sqrt{2x}$ at the point (8,4).

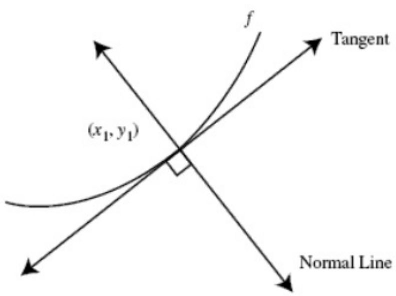
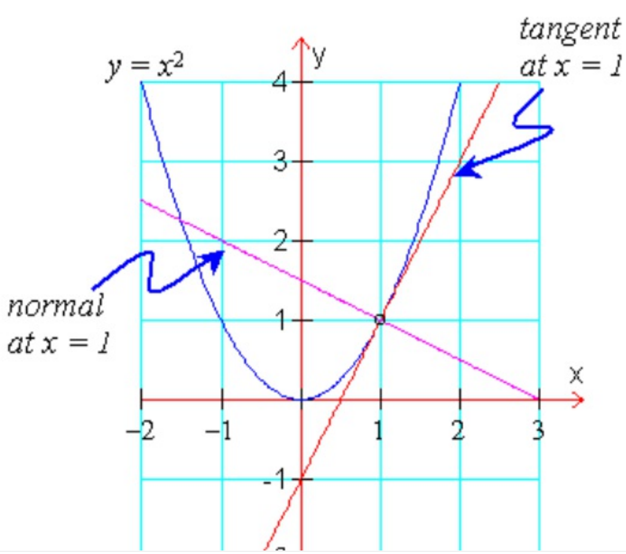


Figure 9.1-10

$$y=(2x)^{1/2}$$
$$y' = 1/2\ (2x)^{-1/2} * 2 \qquad \text{(chain rule)}$$
$$y' = (2x)^{-1/2}$$
$$y' = \frac{1}{\sqrt{2x}}$$



Plug in x=8

$y'(8) = 1/4$ (slope of tangent line)

Think back to geometry....perpendicular lines have <u>opposite reciprocal slopes</u>

So if tan line has m=1/4, then normal line has m= -4/1

Point Slope Form:

$y-y_1=m(x-x_1)$ $\longrightarrow$ $y-4 = -4\ (x-8)$

Good afternoon: welcome back! We will start class with notes when the bell rings

Tutoring Change!!!

~~Tuesday afternoon~~ Wednesday afternoon

What are we covering this quarter?

- Finishing differentiation: related rates, optimization
- Antiderivatives (Indefinite Integration): finding the function families which have a given a derivative
- Definite integration: accumulated change, area, Riemann Approximiations, the Fundamental Theorem of Calculus
- Using integration: motion revisited, volumes of solids

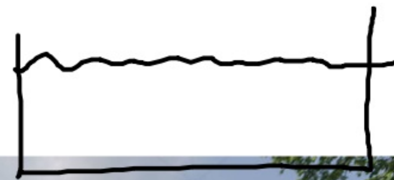
Related Rates Blowing up a balloon

Constant air volume

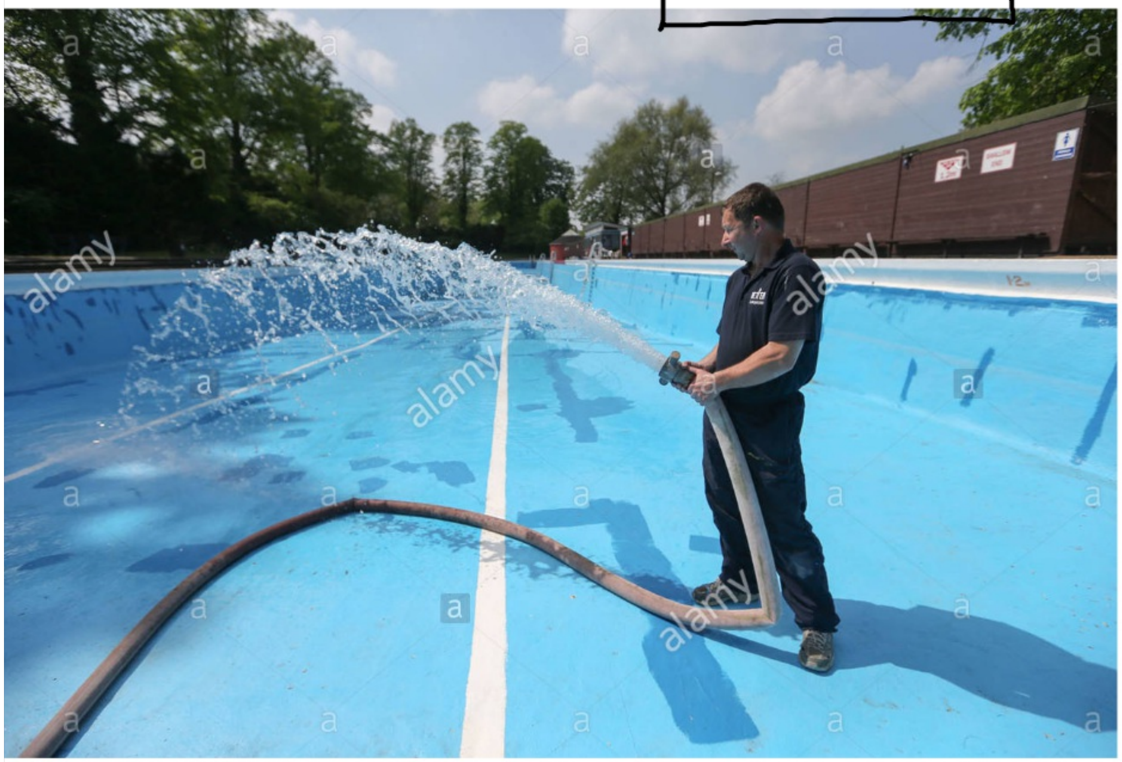
Is the surface area growing at a constant rate??

(no, but the rates
are related to each other)

[redacted]



rate of water coming out of hose



rate of water depth

related, but not equal

Understanding "derivative with respect to"

Suppose x and y are functions of time t. Find dy/dt for

$x(t)$ $y(t)$

$$\frac{d}{dt}\left[3x^2+5y^2=12\right]$$

Implicit differentiation, doing d/dt to both sides instead of d/dx

$$3\cdot 2x\cdot\frac{dx}{dt} + 5\cdot 2y\cdot\frac{dy}{dt} = 0$$

$$6x\frac{dx}{dt} + 10y\frac{dy}{dt} = 0$$

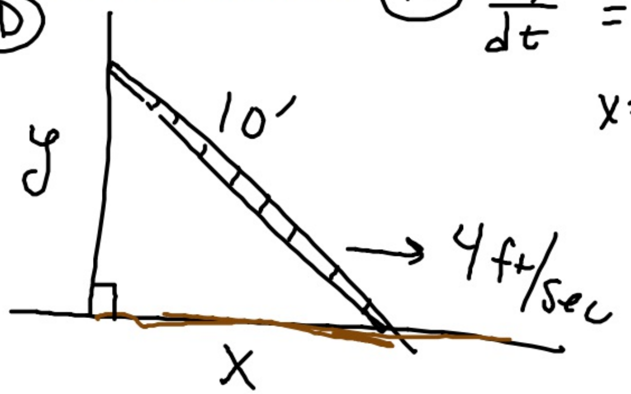
$$\frac{\cancel{10y}\frac{dy}{dt}}{\cancel{10y}} = \frac{-6x\frac{dx}{dt}}{10y} \rightarrow \boxed{\frac{dy}{dt} = \frac{-3x\frac{dx}{dt}}{5y}}$$

The Falling Ladder Problem DREDS

A 10 foot ladder is leaning against a wall. It has rained and turned the ground into a muddy, slippery mess. A gust of wind of constant velocity nudges the ladder so that the base of the ladder slides away from the wall at a rate of 4 feet per second.

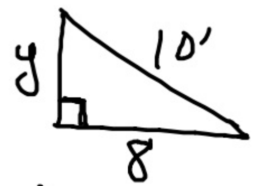
How fast is the top of the ladder sliding down the wall when the base of the ladder is 8 feet from the wall?

D



R $\frac{dx}{dt} = 4$ $\quad \frac{dy}{dt} = ?$

$x = 8$



$8^2 + y^2 = 10^2$

$y = 6$

E $(x^2 + y^2 = 10^2)\frac{d}{dt}$

D $2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

S $2 \cdot 8 \cdot 4 + 2 \cdot 6 \cdot \frac{dy}{dt} = 0$

$64 + 12\frac{dy}{dt} = 0$

$12\frac{dy}{dt} = -64$

$\frac{dy}{dt} = -5.333$ ft/sec.

D Diagram

R Rates (label given information, needed information)

E Equation (geometry formula, usually. Be sure needed info is only unknown)

D Differentiate (with respect to TIME)

S Substitute, then solve

A spherical balloon is being inflated by a pump at a rate of 3 cubic centimeters per minute. At what rate is the radius of the balloon changing when the diameter of the balloon is 4 cm?

D

$$V = \frac{4}{3}\pi r^3$$

R

$$\frac{dV}{dt} = 3 \quad d = 4 \rightsquigarrow r = 2$$

$$\frac{dr}{dt} = ?$$

E

$$V = \frac{4}{3}\pi r^3$$

D

$$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} r^2 \cdot \frac{dr}{dt}$$

✓ ✓ ?

S

$$3 = 4\pi \cdot \cancel{2^2}^{\,4} \cdot \frac{dr}{dt} \checkmark$$

$$\frac{3}{16\pi} = \frac{dr}{dt}$$

~~$SA = 4\pi r^2$~~

~~$A = \pi r^2$~~

~~$C = 2\pi r$~~

A spherical snowball melts such that its radius decreases at a rate of 2 in/min. At what rate is the volume of the snowball changing when the radius is 3 inches?

D

R

$$\frac{dr}{dt} = -2$$

$$\frac{dV}{dt} = ? \qquad r = 3$$

E

$$V = \frac{4}{3}\pi r^3$$

D

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

S

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

? ✓ ✓

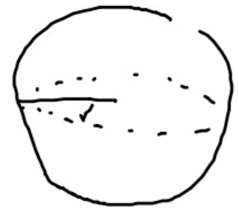
$$\frac{dV}{dt} = 4\pi \cdot 3^2 \cdot -2$$

$$\boxed{\frac{dV}{dt} = -72\pi \ \text{in}^3/\text{min}}$$

A balloon is being inflated such that its radius grows at a rate of 2 in per minute. How fast is the surface area changing when the balloon's volume is $32/3\,\pi$ in^3?

(Not done in class)

(D)



$\leftarrow$ 2

(R) $\frac{dr}{dt} = 2 \quad V = \frac{32}{3}\pi$

$\frac{dS}{dt} = ?$

(E) $S = 4\pi r^2$ [Surface Area of Sphere formula

(D) $\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$ ✓

?

Not given!

use $V = \frac{32}{3}\pi$

$\frac{32}{3}\pi = \frac{4\pi}{3} r^3$

$32\pi = 4\pi r^3$

$8 = r^3$

$\underline{\underline{2 = r}}$

(S)

$\frac{dS}{dt} = 8\pi \cdot 2 \cdot 2$

$$\boxed{\frac{dS}{dt} = 32\pi \text{ in}^2/\text{min}}$$

A ripple forms in a pond. It is expanding such that its radius grows by 2 inches per second. How fast is the area growing when the ripple's circumference is 32π in?

E $A = \pi r^2$

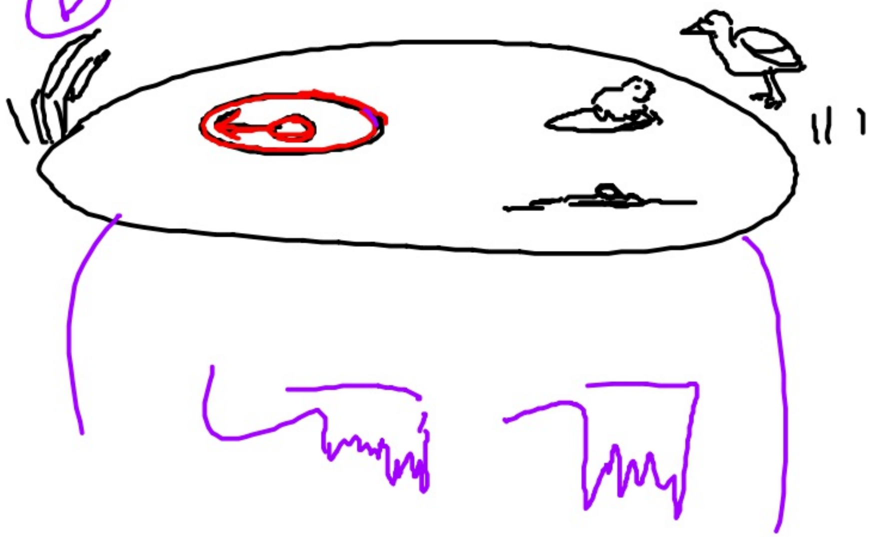
R $\frac{dr}{dt} = 2$

$C = 32\pi = 2\pi r$

$r = 16$

$\frac{dA}{dt} = ?$

D

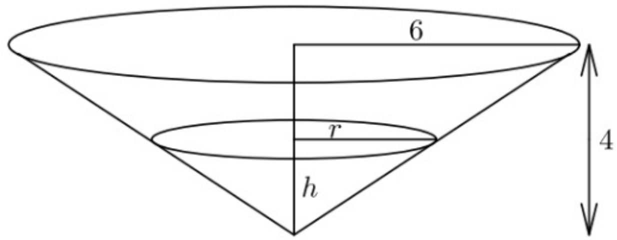


$A = \pi r^2$

$\frac{dA}{dt} = \boxed{2\pi r} \frac{dr}{dt}$

$= 32\pi \cdot 2$

$\boxed{64\pi \ \text{in}^2/\text{s}}$



7. The conical reservoir shown above has diameter 12 feet and height 4 feet. Water is flowing into the reservoir at the constant rate of 10 cubic feet per minute. At the instant when the surface of the water is 2 feet above the vertex, the water level is rising at the rate of

(to do Friday)

A) 0.177 ft per min

B) 0.354 ft per min

C) 0.531 ft per min

D) 0.708 ft per min

E) 0.885 ft per min

p. 153

#15-18, 21, 24a

Don't forget calcchat