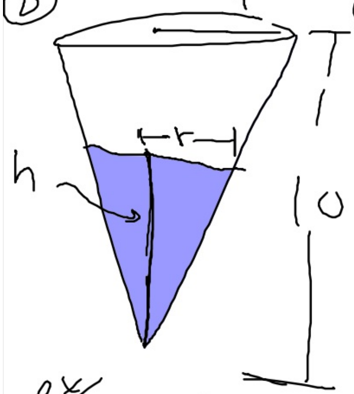
A water tank in the shape of a right circular cone has a height of 10 feet. The top rim of the tank is a circle with a radius of 4 feet. If water is being pumped into the tank at the rate of 2 $\boxed{\text{cubic feet}}$ per minute, what is the rate of change of the water depth, in feet per minute, when the depth is 5 feet?

D

R

E



$\frac{dh}{dt} = ?$

$h = 5$

$\frac{dV}{dt} = 2$

$V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \left(\frac{2}{5}h\right)^2 h$

$\frac{\pi}{3} \cdot \frac{4}{25} \cdot h^2 \cdot h$

$\frac{4}{10} = \frac{r}{h}$

$4h = 10r$

$\frac{2h}{5} = r$

D

$\frac{d}{dt}\left(V = \frac{4\pi}{75} h^3\right)\frac{d}{dt}$

$\frac{dV}{dt} = \frac{4\pi}{75} \cdot 3h^2 \cdot \frac{dh}{dt}$

$2 = \frac{12\pi}{\cancel{75}_{3}} \cdot \frac{\cancel{25}}{1} \cdot \frac{dh}{dt}$

$2 = 4\pi \frac{dh}{dt}$

$\frac{1}{2\pi} = \frac{dh}{dt}$

ft/min.

ex

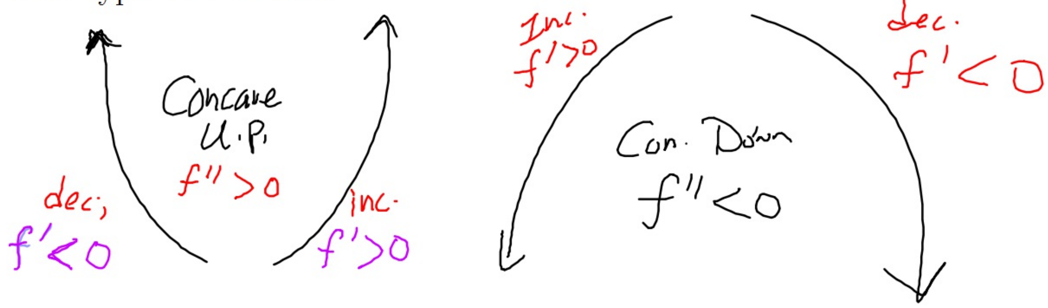
$y = 5x^2$

$y' = 5 \cdot 2x$

$10x$

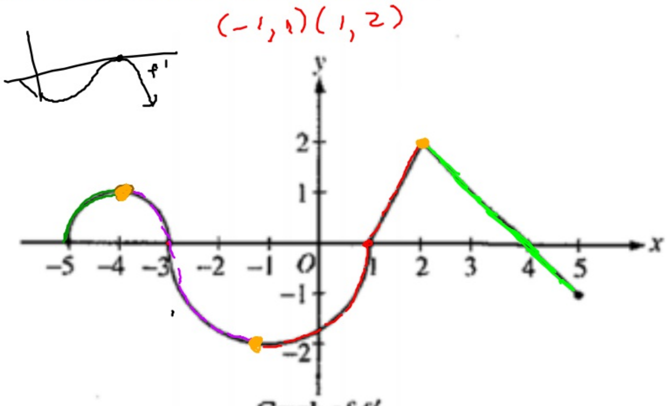
Curve Sketching (will return to related rates right before asssessment)

Four types of curvature



What the first derivative tells you

(-1, 1) (1, 2)



Graph of f'

where is f...

increasing?

(-5, -3) (1, 4)

decreasing?

(-3, 1) (4, 5)

rel max?

$x = -3$ $x = 4$

rel min?

$x = 1$

concave up?

(-5, -4) (-1, 2)

concave down?

(-4, -1) (2, 5)

inflection points?

$x = -4, -1, 2.$

increase, c.u.

(-5, -4) (1, 2)

increase, c.d.

(-4, -3) (2, 4)

decrease, c.u.

(-1, 1)

decrease, c.d.

(-3, -1) (4, 5)

Find: §3.6 in textbook

- domain
- horizontal asymptotes using limits (plot)
- vertical asymptotes using limits (plot)
- x-intercept, y-intercept (plot)
- intervals of increase and decrease
- relative extrema (coordinates, plot them)
- inflection points (coordinates, plot them)
- intervals of Conc. Up, Conc. Down
- Combine number lines to find 4 types of curves

$$f(x) = \frac{x-1}{x+2}$$

Domain: set denom $= 0$.

$x+2=0$

$x=-2$. All x s.t. $x \neq -2$.

H.A.

$$\lim_{x\to\infty} \frac{x-1}{x+2} = \frac{\infty - 1}{\infty + 2} = \frac{\infty}{\infty} = 1 \quad y = 1$$

(Same degree, so ratio of leading coeff.)

V.A.

$$\lim_{x\to -2} f(x) = \infty$$ [Where denom $=0$ unless you can cancel it out.]

$x=-2$

x-int set $y=0$

$$0 = \frac{x-1}{x+2}$$

$0 = x-1$

$x=1$ $(1,0)$

y-int set $x=0$

$$y = \frac{0-1}{0+2} = -\frac{1}{2}$$

$(0, -\frac{1}{2})$

Increase / Decrease

$$f'(x) = \frac{1\cdot(x+2)-(x-1)\cdot 1}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

Find C.N.

$$\frac{3}{(x+2)^2} = 0$$

C.N: = where $f'(x) = 0$ OR undefined.

$\underline{x = -2}$
C.N.

f' +   +

x -2

f Inc.   Inc.

- Inc. $(-\infty, -2)$ $(-2, \infty)$
- Dec: nowhere

Rel. Extrema

See number line above. no sign change $\Rightarrow$ no rel max/min

Inflection Points / Concavity

$$f'(x) = \frac{3}{(x+2)^2}$$

f'' ↓

$$f'' = \frac{0\cdot\cancel{(x+2)^2} - 3\cdot 2(x+2)^1}{(x+\ )^4} = \frac{-6(x+2)^1}{(x+2)^{\cancel{4}\,3}} = \frac{-6}{(x+2)^3}$$

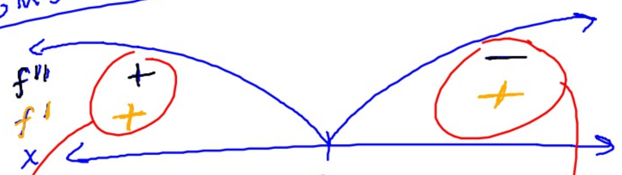
Terrace points where $f''(x) = 0$ or undefined.

$$\underbrace{x = -2}_{T.P.}$$

f''	$f''(-1000) = +$		$f''(0) = -$	
x	-1000	-2	0	
f	C.U.		C.D.	

Sign change ⇒ Inflection point @ $\underline{x = -2}$

Number lines Combined



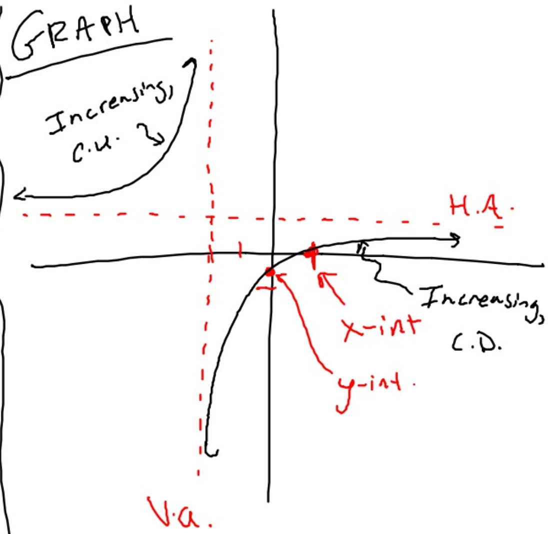
Critical # → −2

Terrace Pts: → −2

Increasing,
Concave up
looks like:

Increasing,
concave down
(looks like:)

GRAPH



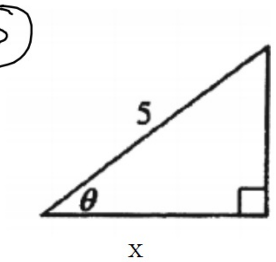
Check with Calculator:

Whew.
Not bad!

Related Rates, again.

In the diagram below, theta is increasing at a constant rate of 2 radians per minute. At what rate is x increasing in units per minute when x is 3[redacted] units long?

(D)



(R) $\frac{d\theta}{dt} = 2$

$x = 3$

$\frac{dx}{dt} = ?$

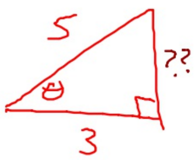
(E) $\cos(\theta) = \frac{x}{5}$

SOHCAHTOA

(D) $-\sin(\theta)\frac{d\theta}{dt} = \frac{1}{5}\cdot\frac{dx}{dt}$

what?? ✓ ?

Need θ... or sin(θ), to be precise.



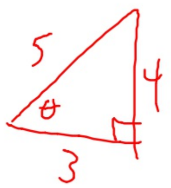
$\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{??}{5}$

pythagorean theorem!

$3^2 + ?^2 = 5^2$

$\Big\}$ math

$? = 4$



so $\boxed{\sin(\theta) = \frac{4}{5}}$

(S)

$-\sin(\theta)\frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$

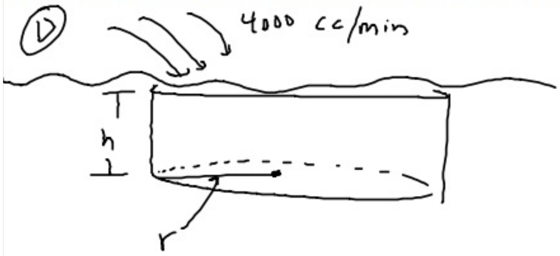
$-\frac{4}{5}\cdot 2 = \frac{1}{5}\cdot\frac{dx}{dt}$

$5\left(-\frac{8}{5} = \frac{1}{\cancel{5}}\frac{dx}{dt}\right)\cancel{5}$

-8 units/min $= \frac{dx}{dt}$

Dyed salt water is being poured at a rate of 4000 $\overbrace{\text{cm}^3}^{\text{vol}}/\overbrace{\text{min}}^{\text{time}}$ into a fresh water tank forming a cylinder whose radius and height are changing with time. When the cylinder is 400cm in diameter and 0.5 cm thick, the radius is increasing at a rate of 1 cm/min. How quickly is the thickness of the cylinder changing? $= \frac{dh}{dt}$

D



R

$\frac{dh}{dt} = ?$

$\frac{dr}{dt} = 1 \text{ cm/min}$

Diameter = 400cm
height/thickness = 0.5cm

$r = 200$

$h = 0.5$

E

$V_{cyl} = \pi r^2 h$ } prod. rule

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \cdot \frac{dh}{dt}$$

4000   200   1   0.5   200   ?

yay! no pesky terms to find.

S

$$4000 = 2 \cdot \pi \cdot \overbrace{200}^{r} \cdot \overbrace{1}^{\frac{dr}{dt}} \cdot \overbrace{0.5}^{h} + \pi \cdot \overbrace{200}^{r}{}^2 \cdot \frac{dh}{dt}$$

$$4000 = \cancel{628.319} + 200^2 \pi \frac{dh}{dt}$$

$$-628.319 \qquad -\cancel{628.319}$$

$$\frac{3371.681}{200^2\pi} = \frac{200^2 \cancel{\pi} \cdot \frac{dh}{dt}}{\cancel{200^2 \pi}}$$

$$\boxed{0.027 \text{ cm/min} = \frac{dh}{dt}}$$