Good afternoon: warm up in notebooks

Show why $\lim_{x\to 4} \frac{|4-x|}{-(x-4)}$ does not exist.

$$\lim_{x\to 4} \frac{|4-x|}{4-x}$$

$4 - x = 0 \implies x = 4$

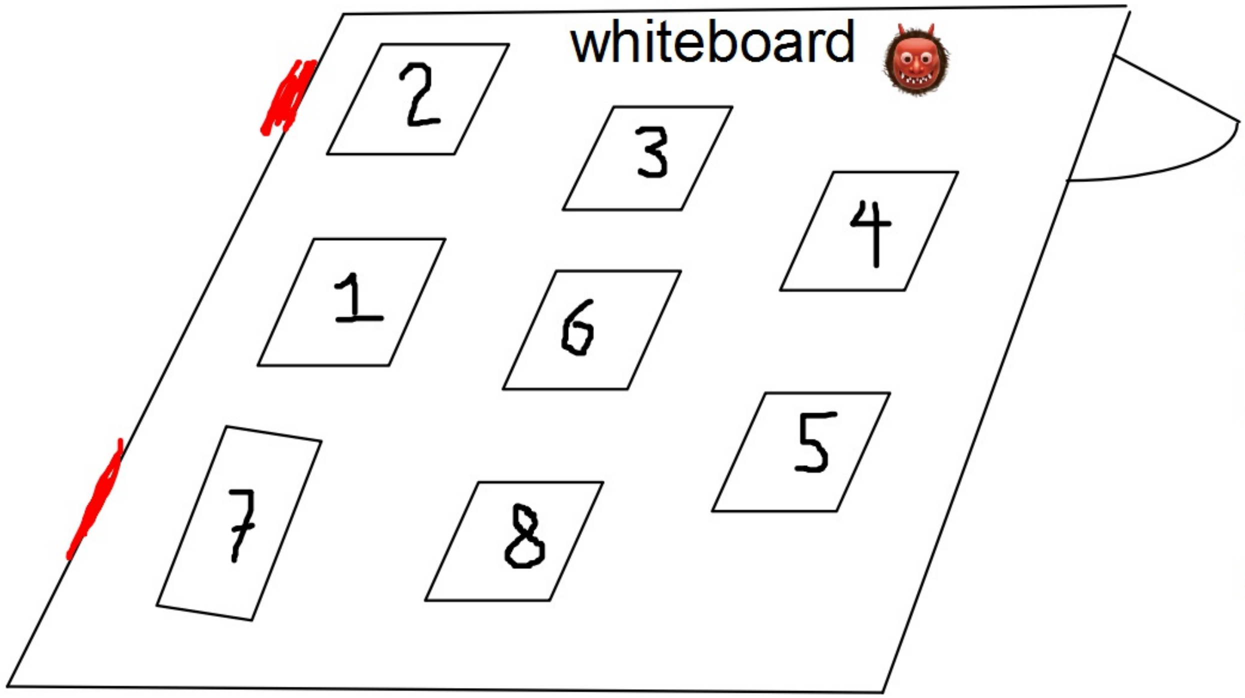
$$f(x) = \begin{cases} \cancel{\frac{4-x}{4-x}}, & x < 4 \\ \cancel{\frac{-(4-x)}{4-x}}, & x > 4 \end{cases} = \begin{cases} 1, & x < 4 \\ -1 & x > 4 \end{cases}$$

$\lim_{x\to 4^-} f(x) = 1$

$\lim_{x\to 4^+} f(x) = -1$

dne

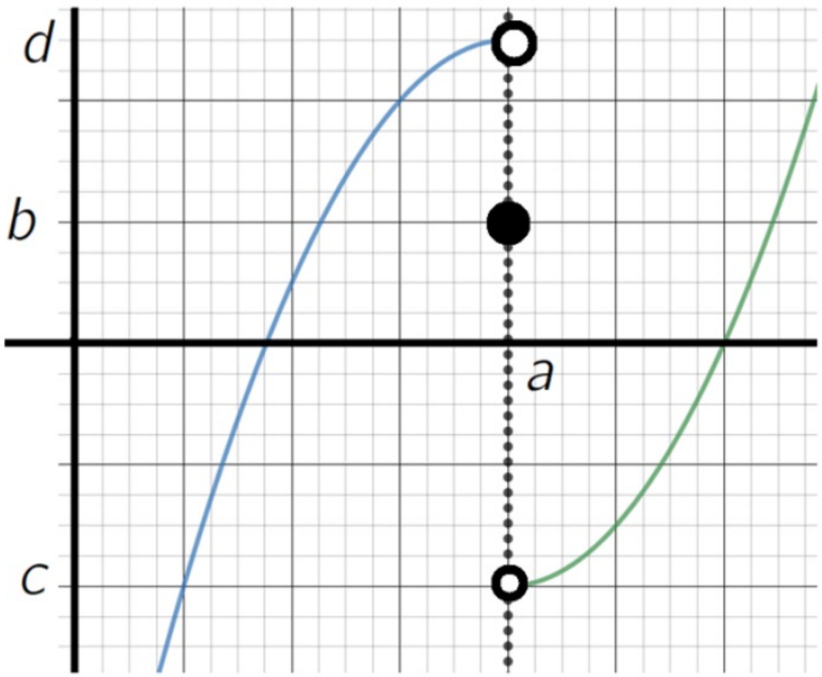
Reminders:
- tutoring 4-5p - next assessment: 9/8
- retakes in DS any day but Weds

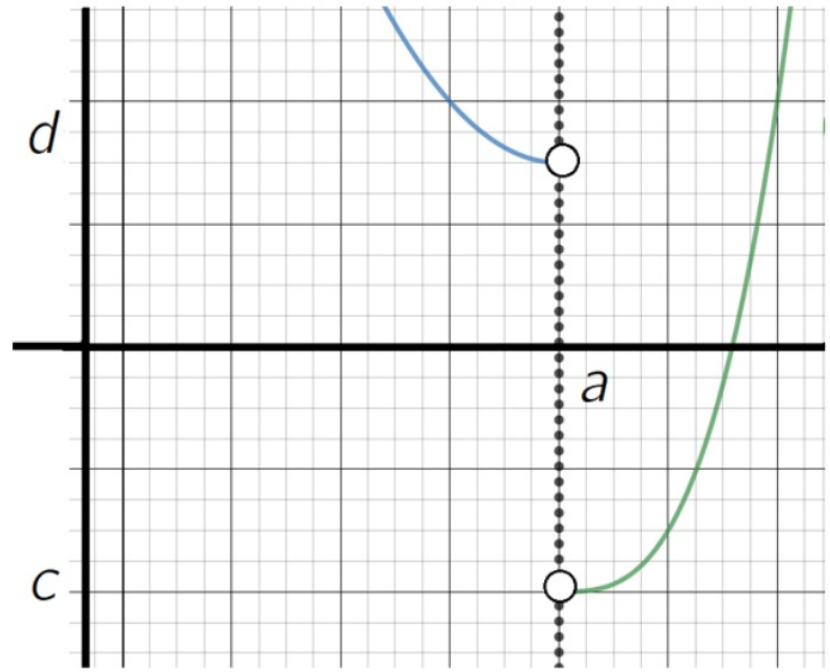


Look over your group's assigned number, be sure all agree with wording and notation for all 3 parts of your problem.

Elect 1 person to write work on the board

- n . ?





n o bridge

b. $\lim_{x \to a^-} A(x) = d$ a Not Continuous

$A(x) = b$

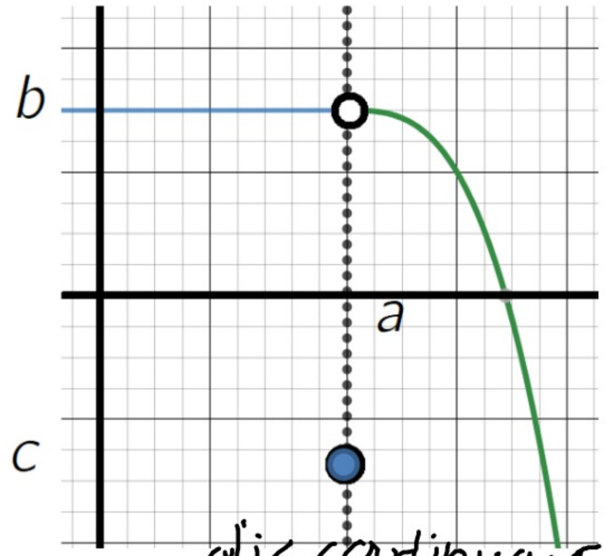
$\lim_{x \to a^+} A(x) = c$

c. Jump

B $\lim_{x \to a^-} M(x) = d$ $\lim_{x \to a^+} M(x) = C$ $M(a) = \phi$

C. Jump

3. University of Georgia: $G(x)$



discontinuous

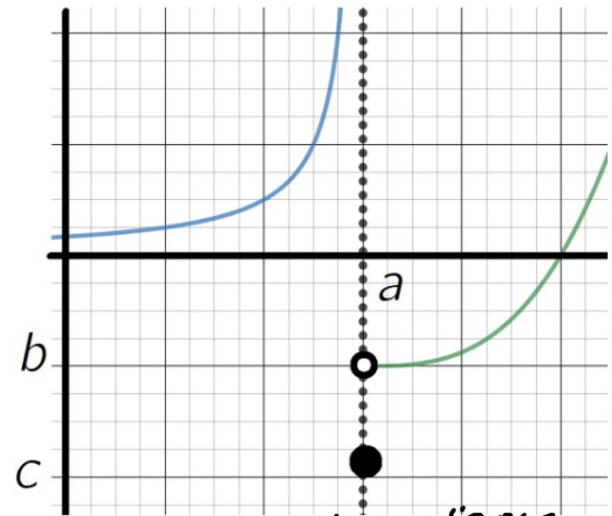
$\lim_{x \to a^-} G(x) = b$

$\lim_{x \to a^+} G(x) = b$

$G(a) = c$

removable

4. University of Chicago $C(x)$



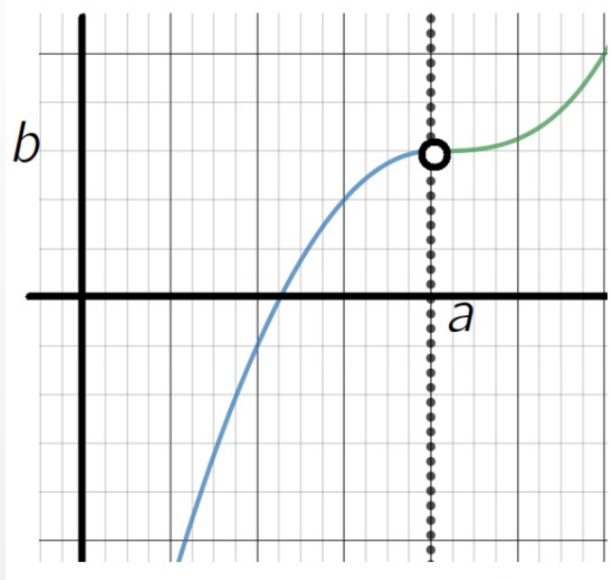
discontinous

$\lim_{x \to a^+} C(x) = b$ $C(a) = c$

$\lim_{x \to a^-} C(x) = \infty$

infinite

5. University of North Carolina: $N(x)$



discontinuous

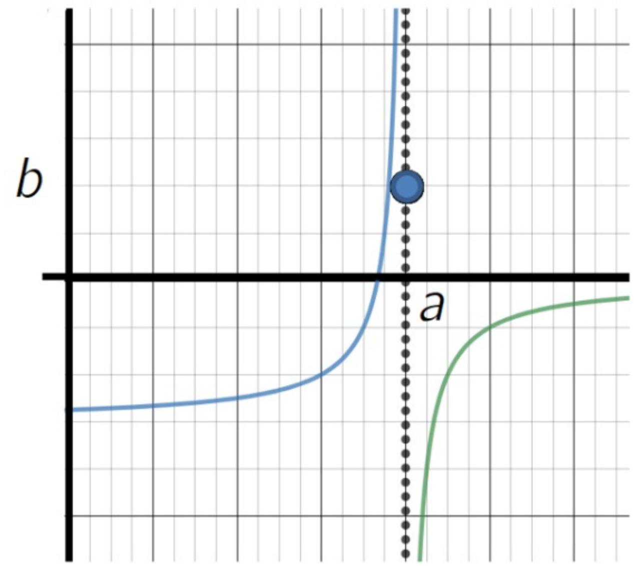
$\lim_{x \to a^-} b$

$\lim_{x \to a^+} b$

$n(a) = ?$

removable

6. University of Tennessee: $T(x)$



discontinuous

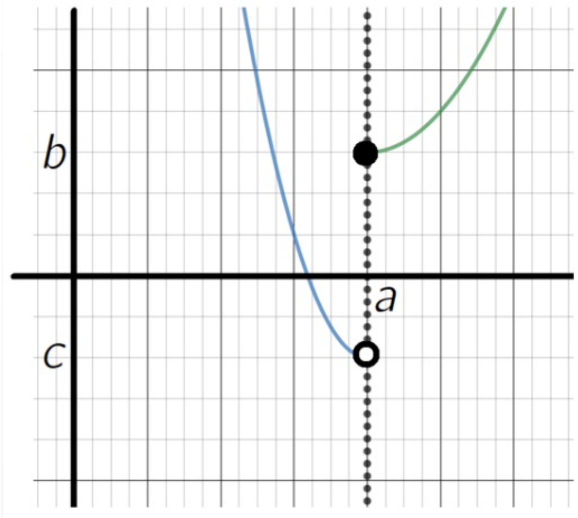
infinite

vertical asymptote

$T(a) = b$

$\lim_{x \to a^-} T(x) = \infty$ $\lim_{x \to a^+} T(x) = -\infty$

7. Stanford University: $S(x)$

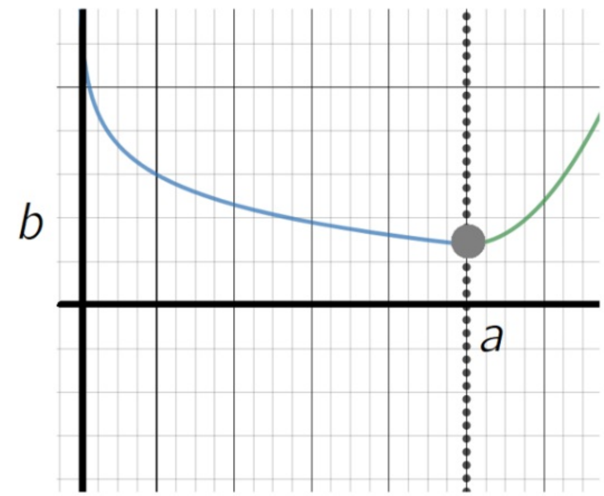


jump.

$\lim_{x \to a^-} s(x) = c$ $\lim_{x \to a^+} s(x) = b$

$s(a) = b$

8. Duke University: $D(x)$



$\lim_{x \to a^+} d(a) = b$

$\lim_{x \to a^-} d(a) = b$

$D(a) = b$

Continuous

Definition of Continuity at a Point

iff $\rightleftarrows$

A function *f(x)* is continuous at a point x=a if and only if

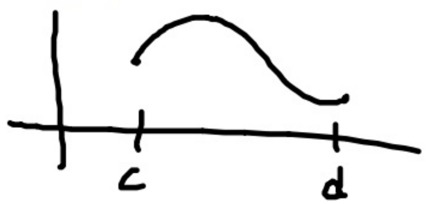
- $$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

Alt:

- $$\lim_{x \to a} f(x) = f(a)$$

Continuity on an Interval

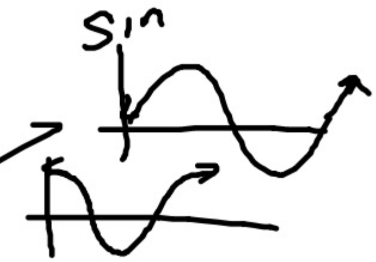
A function is continuous on an interval if it is continuous at every point *in* the interval

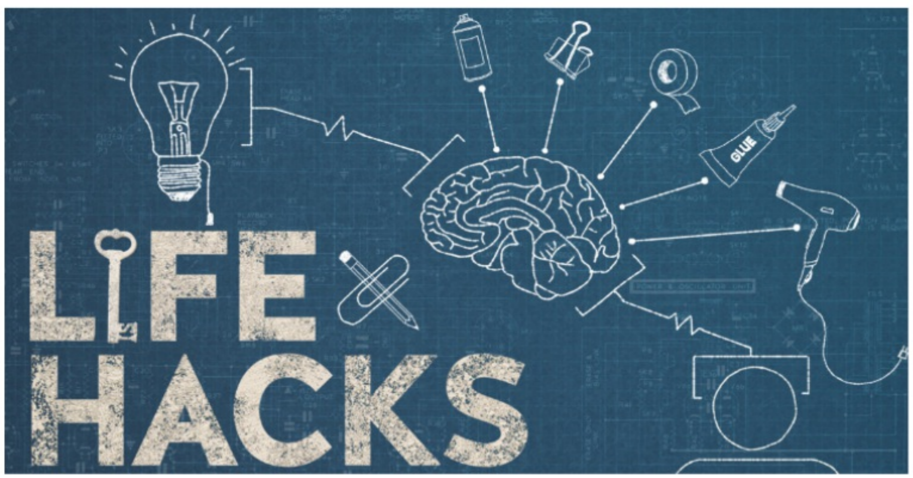


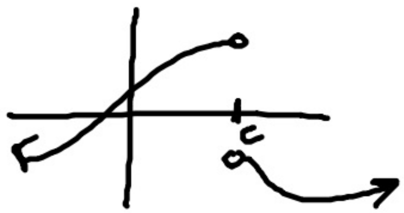
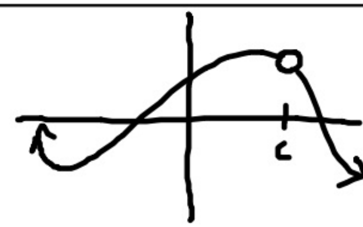
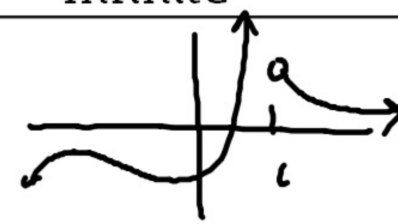
A function is continuous everywhere except where it is not

Save time:

- polynomial functions are always continuous everywhere
- piecewise functions often yield "jump" discontinuities
- sin(x) and cos(x) are always continuous everywhere
- rational functions often have v.a. or holes ("removable") discontinuities





	Jump	Removable	Infinite
Graphically	 	 	
Algebraically	$\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x)$ but both $\in \mathbb{R}$ (both are finite)	$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$ but ... $f(c) \neq \lim_{x\to c} f(x)$	either: $\lim_{x\to c^-} f(x) = \pm\infty$ or $\lim_{x\to c^+} f(x) = \pm\infty$
Notes	- often piecewise	- limit exists! - often results from cancelling by factoring.	- Vertical Asymptote - Rational functions leading to div. by zero.

Discuss the continuity of f(x). Classify any discontinuities and justify with limits.

test prep!

$$f(x)=\frac{x+1}{2x^2-6x-8}$$

x = -1 remov.?

Factor & Cancel: $\frac{x+1}{2(x^2-3x-4)} = \frac{\cancel{x+1}}{2(x-4)\cancel{(x+1)}} = \frac{1}{2(x-4)}$

x = 4 Infinite?

Justify

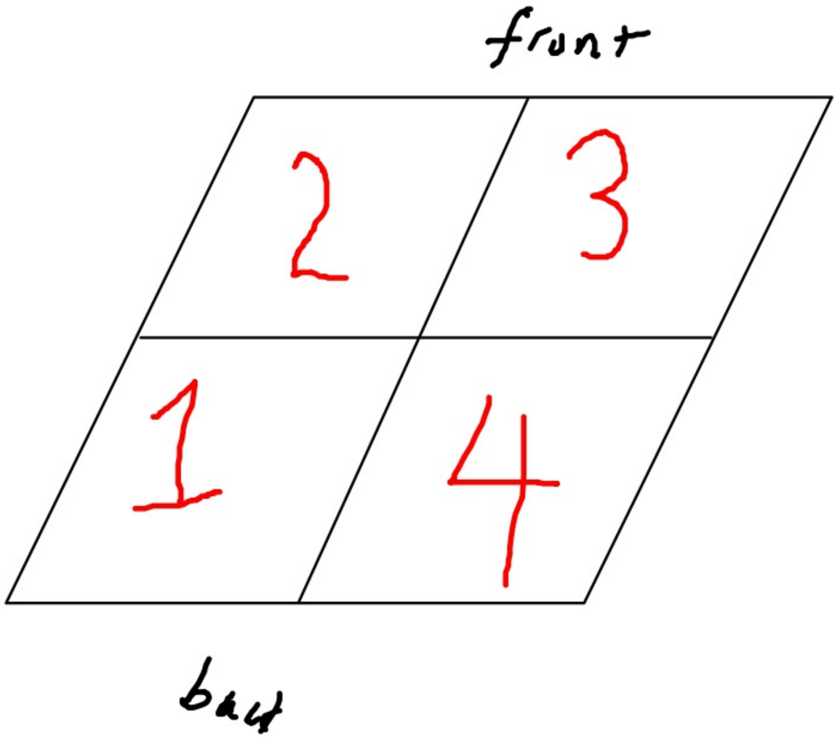
x = -1

$$\lim_{x\to -1} \frac{1}{2(x-4)} = \frac{1}{2(-1-4)} = -\frac{1}{10} \Longrightarrow x=-1 \text{ is remov.}$$

x = 4

$$\lim_{x\to 4^+} \frac{1}{2(x-4)} = \frac{1}{2(4^+-4)} = \frac{1}{2(0^+)} = \frac{1}{0^+} = \infty \Longrightarrow x=4 \text{ inf. disc.}$$

f(x) is cont. everywhere but
x = -1 (remov.) & x = 4 (inf.)



Each person does their own
problem using private time

Then person 1 shares findings
with group, others listen, write, ask q's

Then person 2 shares, etc.

Discuss the continuity of the function. Classify any discontinuities and justify classifications with limits.

1 $f(x)=\begin{cases} x+\frac{1}{2}, & x<\frac{1}{2} \\ 0, & x\geq\frac{1}{2} \end{cases}$

2 $f(x)=-\frac{x^2-3x}{x}$

3 $f(x)=\frac{x^2}{3x-9}$

4 $f(x)=\frac{x-2}{x^2-x-2}$

1 $f(x) = \begin{cases} x + \frac{1}{2}, & x < \frac{1}{2} \\ 0, & x \geq \frac{1}{2} \end{cases}$

linear, cont. everywhere.

Linear, cont. everywhere

Do they meet?!

@ $x = \frac{1}{2}$?!

$$\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} \left(x + \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2} = 1$$

$$\lim_{x \to \frac{1}{2}^+} f(x) = \lim_{x \to \frac{1}{2}^+} 0 = 0 \qquad 0 \neq 1$$

So, Jump!

$f(x)$ is cont. everywhere except $x = \frac{1}{2}$ (Jump)

2 $f(x) = -\frac{x^2-3x}{x} = -\frac{\cancel{x}(x-3)}{\cancel{x}} = x-3$

$x=0$ remov.?

Notice! No denominator, so no v.a/inf. disc. to worry about

Justify

$x=0$? $\lim_{x\to 0} f(x) = \lim_{x\to 0} x-3 = -3$

Finite! So, Removable!

f(x) is cont. everywhere except x=0 (removable).

3 $f(x) = \dfrac{x^2}{3x-9} = \dfrac{x^2}{3(x-3)}$ ← Notice! No need to test $x=0$. 0 can be in numerator.

$\rightarrow x=3$ infinite?

Justify

Is $x=3$ infinite disc.?

$$\lim_{x \to 3^+} \frac{x^2}{3(x-3)} = \frac{(3^+)^2}{3(3^+-3)} = \frac{9^+}{3(0^+)} = \frac{9^+}{0^+} = \infty$$

So yes! it is.

$f(x)$ is continuous everywhere except $x=3$ (infinite discontinuity)

4 $f(x) = \frac{x-2}{x^2-x-2} = \frac{\cancel{x-2}}{\cancel{(x-2)}(x+1)} = \frac{1}{x+1}$

is $x=-1$ infinite?

is $x=2$ removable?

Justify

$x=2$: $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{1}{x+1} = \frac{1}{3}$

finite value, so yes, remov.

$x=-1$ $\lim_{x\to -1^-} \frac{1}{x+1} = \frac{1}{-1^-+1} = \frac{1}{0^-} = -\infty$ yes, inf. disc @ $x=-1$

f(x) is continuous everywhere except $x=2$ (removable) and $x=-1$ (infinite).

HW: p. 80 #39-57 (multiples of 3)

For each:

a. sketch a graph (use calc or desmos.com)

b. find and classify discontinuities algebraically (graph can help you find places to test with limits)

~~#61-65~~

~~Follow book instructions for this part~~