AP Calculus

From the back table get:

- white board
- marker
- eraser rag
- voting egg

Then enter in your PIN

False

[redacted] **True** or **False**. As x increases to 100, $f(x) = 1/x$ gets closer and closer to 0, so the limit as x goes to 100 of $f(x)$ is 0. Be prepared to justify your answer.

$$\lim_{x\to 100} \frac{1}{x} = \frac{1}{100} \qquad \lim_{x\to\infty} \frac{1}{x} =$$

C

[redacted] The statement "Whether or not $\lim_{x \to a} f(x)$ exists, depends on how $f(a)$ is defined," is true

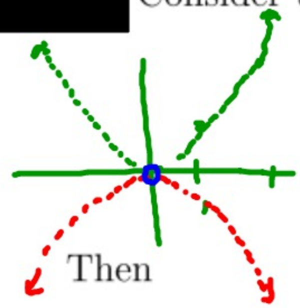
(a) sometimes

(b) always

(c) never

C

[redacted] Consider the function



$$f(x) = \begin{cases} x^2 & x \text{ is rational, } x \neq 0 \\ -x^2 & x \text{ is irrational} \\ \text{undefined} & x = 0 \end{cases}$$

Then

(a) there is no a for which $\lim_{x \to a} f(x)$ exists

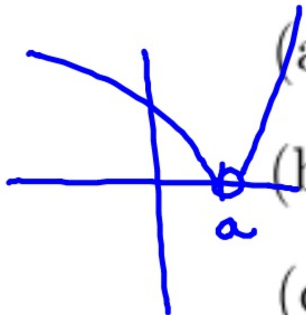
(b) there may be some a for which $\lim_{x \to a} f(x)$ exists, but it is impossible to say without more information

(c) $\lim_{x \to a} f(x)$ exists only when $a = 0$

(d) $\lim_{x \to a} f(x)$ exists for infinitely many a

B

If a function f is not defined at $x = a$,



(a) $\lim_{x \to a} f(x)$ cannot exist

(b) $\lim_{x \to a} f(x)$ could be 0

(c) $\lim_{x \to a} f(x)$ must approach ∞

(d) none of the above.

False

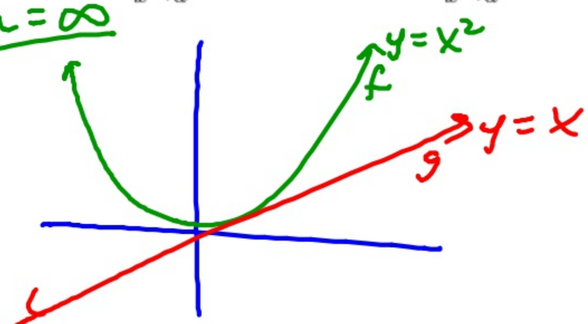
True or **False**. If $\lim\limits_{x\to a} f(x) = \infty$ and $\lim\limits_{x\to a} g(x) = \infty$, then $\lim\limits_{x\to a}[f(x) - g(x)] = 0$.

$a = \infty$

Indet. Form

$\frac{0}{0} \neq 1$

$\infty - \infty$



$\lim\limits_{x\to\infty} x^2 = \infty$

$\lim\limits_{x\to\infty} x = \infty$

$\lim\limits_{x\to\infty} x^2 - x = \infty$

$\infty = 1, \cancel{2}, 3, \cancel{4}, 5, \cancel{6}, 7, \ldots$

$- \quad 2 \quad 4 \quad 6 \quad 8, \ldots$

$1, 3, 5, 7, 9, \ldots$

B

[redacted] A drippy faucet adds one milliliter to the volume of water in a tub at precisely one second intervals. Let f be the function that represents the volume of water in the tub at time t.

(a) f is a continuous function at every time t

(b) f is continuous for all t other than the precise instants when the water drips into the tub

(c) f is not continuous at any time t

(d) not enough information to know where f is continuous.

A:

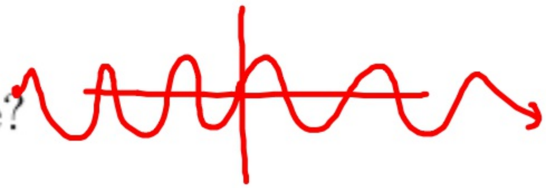
You know the following statement is true: $y = x^n + \dots$

$P \rightarrow Q$

If $f(x)$ is a polynomial, then $f(x)$ is continuous.

$\sim Q \rightarrow \sim P$

Which of the following is also true?



(a) If $f(x)$ is not continuous, then it is not a polynomial.

(b) If $f(x)$ is continuous, then it is a polynomial.

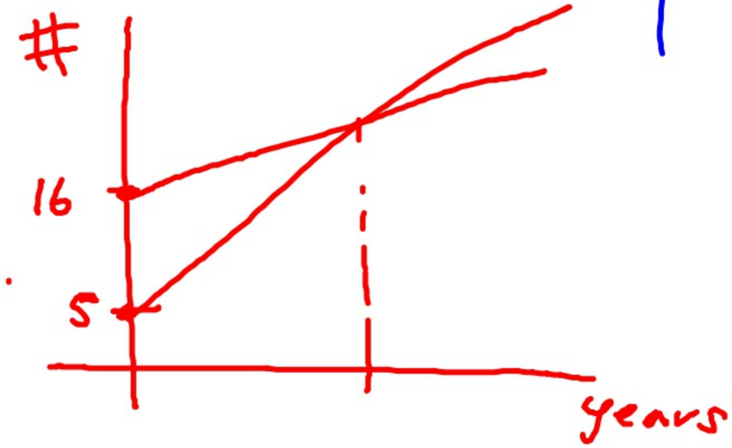
(c) If $f(x)$ is not a polynomial, then it is not continuous.

True or **False**. You were once exactly 3 feet tall.

T

[redacted] **True** or **False**. At some time since you were born your weight in pounds equaled your height in inches.

T



[redacted] **True** or **False**. Along the Equator, there are two diametrically opposite sites that have exactly the same temperature at the same time.

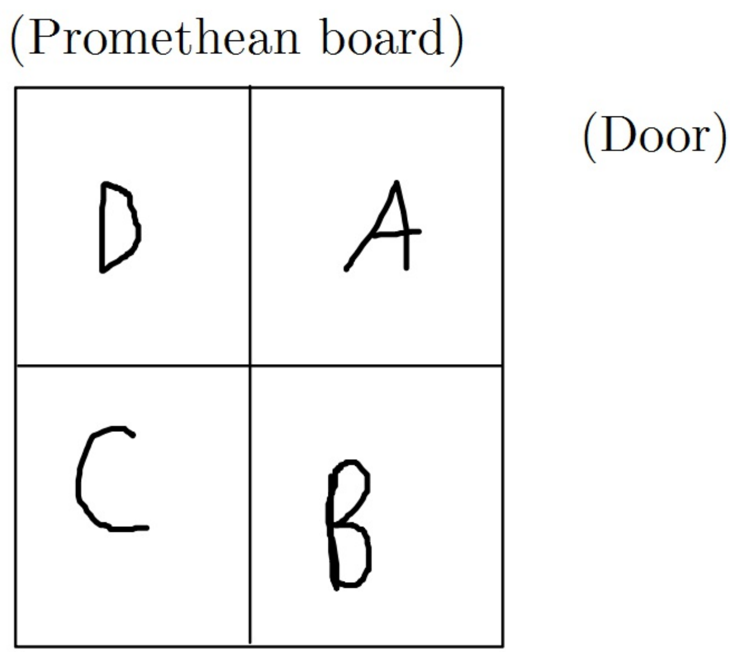
T

Person A: return eggs to crate

Person B: return boards, markers and erasers to bin

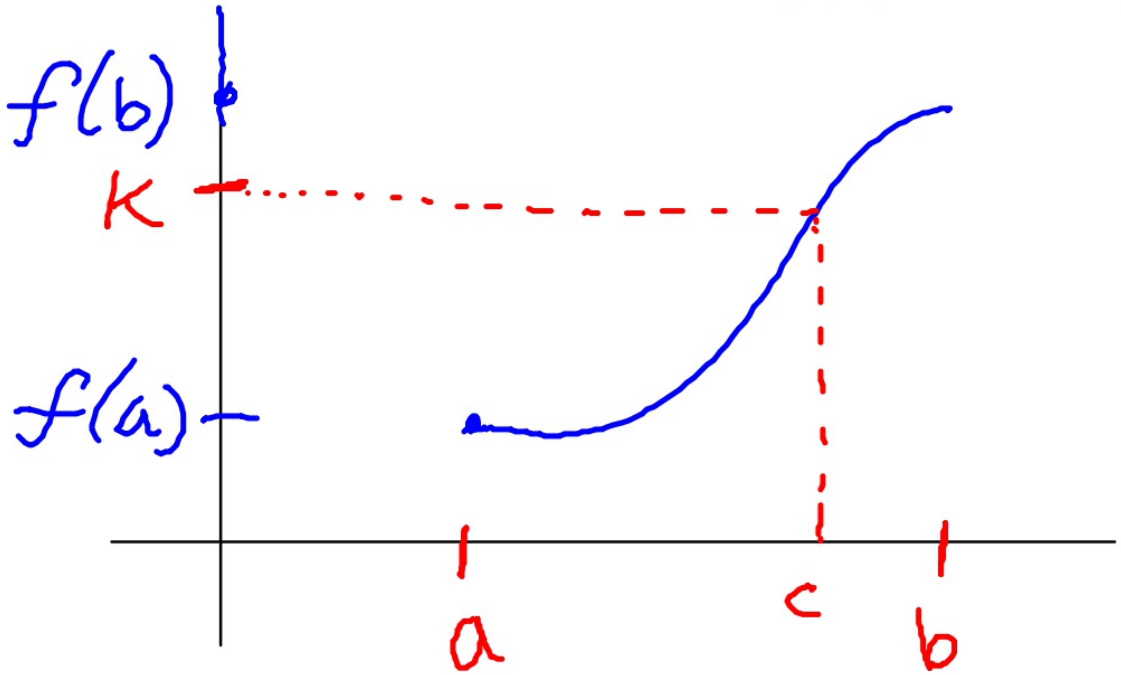
Person C: Get 1 construction paper sheet per person at table

Person D: Get 3 sheets of white paper per ~~group~~ person @ table



The Intermediate Value Theorem

If f is continuous on [a,b] and k is some number between f(a) and f(b), then there exists some number c in [a,b] such that f(c) $= k$



(notes)

IVT is an existence theorem

It only applies to continuous functions!

At some point in the first quarter, the Cavaliers have a score of 26 points.

In the third quarter, their score is 57 points.

So at some point in between, they scored exactly 30 points.

Ex: Use the IVT to prove that $f(x) = x^2 - 6x + 8$ has a zero in $[0,3]$

$f(x)$ is cont., so IVT applies.

$f(0) = 8$

$f(3) = -1$.

Because $f(0) = 8 > f(3) = -1$

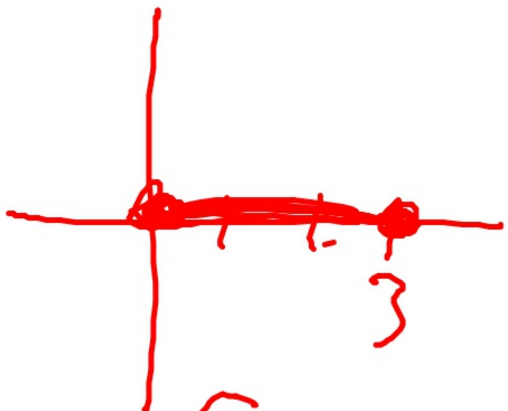
and

$8 > 0 > -1$,

IVT guarantees

some c in $[0,3]$

s.t. $f(c) = 0$



$[0,3]$

Use the IVT to prove that $\sin(x)$ has a root in $[\pi/2, \pi]$

Finding C

SEE NEXT PAGE FOR BETTER EXAMPLE

Find the value(s) of c in [-2,3] guaranteed by the IVT such that f(c)=1

$$f(x) = x^2-5x+6$$

$f(-2)=20$

$f(3)=0$

$x^2-5x+6=\cancel{1}$

$-1 \quad -1$

$Ax^2-5x+5=0$

$B \quad C$

$$\cancel{x}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{5\pm\sqrt{5}}{2}$$

$\cancel{3.618}$

1.382

Find the value of c that is in [0,7] that is guaranteed to exist by the IVT for $f(x)=x^2+6x+1$ such that $f(c)=28$

- $f(x)$ is polynomial, so continuous. Thus, IVT applies.

$f(0) = 1$

$f(7) = 92$

$1 < 28 < 92$, so IVT says c exists in [0,7] such that

$f(c) = 28$

$c^2 + 6c + 1 = 28$

$-28 \quad -28$

$c^2 + 6c - 27 = 0$

$(c+9)(c-3) = 0$

$c = -9$ $\quad$ $c = 3$

Not in [0,7] $\quad$ there it is!

AP Limit Problems

Due Friday