Good afternoon: warm up in notebooks

Use a calculator and a numerical method to approximate the following:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$\left(1+\frac{1}{\infty}\right)^{\infty}$$

$$1^{\infty} = 1 \qquad (1.000\ldots)^{\infty} = e$$

Using $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, calculate:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{3n}$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^{3}$$

$$\left(\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}}\right)^{3}$$

$$e^{3}$$

$$\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{5n}$$

$$\cancel{n}\left(\frac{2}{n}=\frac{1}{m}\right)n$$

$$m\left(2=\frac{n}{\cancel{m}}\right)\cancel{m}$$

$$\lim 2m = \lim_{n\to\infty} n$$

∞ ∞

$n\to\infty$

$m\to\infty$

$$\lim_{n\to\infty}\left(1+\frac{2}{2m}\right)^{5\cdot 2m}$$

$$\left(1+\frac{\cancel{2}}{\cancel{2}m}\right)^{10m}$$

$$\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^{10m} = e^{10}$$

Questions from hw or assessment?

$$f(x) = \frac{2x-3}{4x+1} \quad H.A.?$$

$$\lim_{x \to \infty} \frac{2x-3}{4x+1} = \frac{2\cancel{\infty} - \cancel{3}}{4\cancel{\infty} + \cancel{1}}$$

$$y = \frac{1}{2}$$

5.) $\lim_{x \to 0} \frac{\sin(2x)}{3x}$

$$\lim_{x \to 0} \frac{1}{3} \cdot \frac{\sin 2x}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1}{3} \lim_{x \to 0} \frac{\sin 2x}{x} \cdot \frac{2}{2}$$

$$\frac{1}{3} \lim_{x \to 0} \frac{2 \sin 2x}{2x}$$

$$\frac{1}{3} \cdot \lim_{x \to 0} 2 \boxed{\lim_{x \to 0} \frac{\sin 2x}{2x}}$$

$$\frac{1}{3} \cdot 2 \cdot 1$$

$$\frac{2}{3}$$

i)

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$\frac{x-1}{\cancel{(x-1)}(\sqrt{x}+1)}$$

$$\lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

18.) $\lim\limits_{x \to -\infty} \dfrac{2x - 3}{\sqrt{4x^2 + 2}}$

???

$\sqrt{36} = |6|$

± 6

$\lim\limits_{x \to -\infty} \dfrac{2x}{\sqrt{4x^2}}$

$\lim\limits_{x \to -\infty} \dfrac{2x}{|2x|} = \dfrac{-2\infty}{|-2\infty|} = \dfrac{-2\cancel{\infty}}{2\cancel{\infty}} = -1$

Absolute Value Limits

Abs. Values are piecewise in disguise!!!!

$$\lim_{x \to 1} \frac{x^2 - 1}{|x - 1|}$$

① Set absolute value part = 0.

② Solve. This is the "handoff.

③ Write a piecewise w/ handoff pt.

4. Determine +/- by testing values on either side of handoff

5. Simplify each piece if possible

6. Evaluate the limit

$$\begin{aligned} x - 1 &= 0 \\ +1 \quad & \; +1 \\ x &= 1 \end{aligned}$$

$$\begin{cases} \dfrac{x^2-1}{x-1}, & x > 1 \\ \dfrac{x^2-1}{-(x-1)}, & x < 1 \end{cases}$$

$$\begin{cases} \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}, & x > 1 \\ \dfrac{(x+1)\cancel{(x-1)}}{-\cancel{(x-1)}} \end{cases} = \begin{cases} x+1, & x > 1 \\ -x-1, & x < 1 \end{cases}$$

$$\lim_{x \to 1^+} x+1 = 2$$

$$\lim_{x \to 1^-} -x-1 = -2$$

d.n.e.

$$\lim_{x\to -2+} \frac{-4x-8}{|-x-2|}$$

$x = -2$

$$\begin{cases} \dfrac{-4x-8}{-x-2}, & x < -2 \\ \dfrac{-4x-8}{-(-x-2)}, & x > 2 \end{cases}$$

$$\begin{cases} \dfrac{4\cancel{(-x-2)}}{\cancel{-x-2}}, & x < -2 \\ \dfrac{4\cancel{(-x-2)}}{-\cancel{(-x-2)}} \end{cases}$$

$$\begin{cases} 4, & x < -2 \\ -4, & x > -2 \end{cases}$$

-4

MYTH: Calculus is all about taking limits of more and more complicated functions.

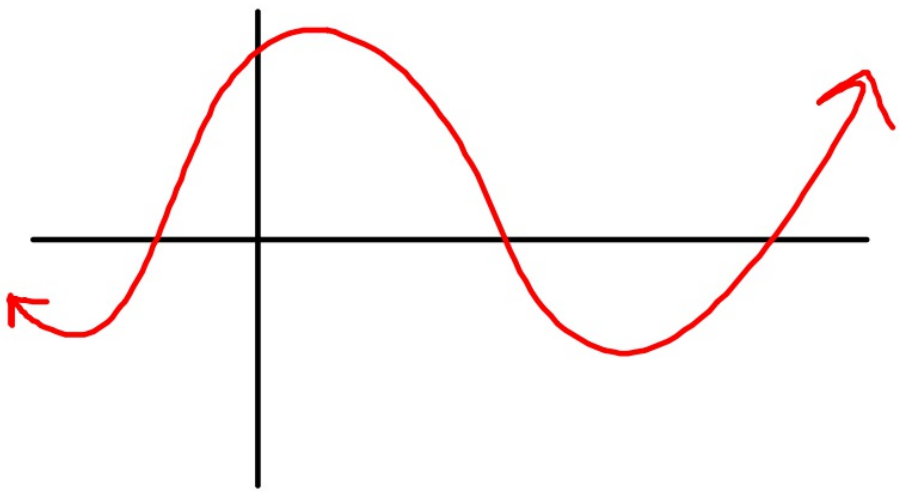
FACT: Calculus is about things that change and how change occurs, on small and big scales.

[redacted]

Our limits toolkit

- direct substitution
- factor/cancel
- rationalize w/ conjugate
- take one-sided limit(s)
- use degrees/growth with infinity
- use special trig rules
- rewrite as piecewise

Continuity



"a function drawn w/o picking up pencil"

HW: finish bridges handout