How to find the area under a curve?

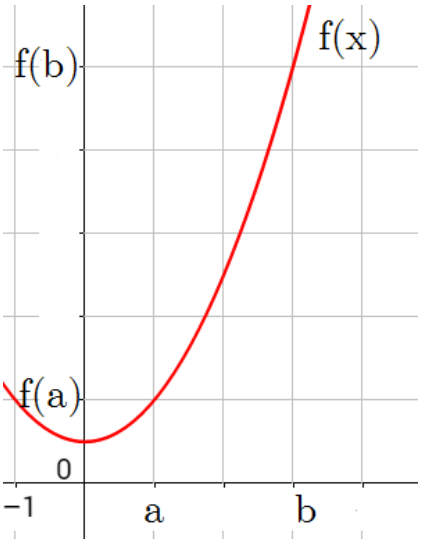
First approximation

Find a method to approximate the area enclosed by $y = x^2 + 1$, the vertical lines x=1 and x=3, and the x-axis.
(This is called 'the area under the curve')

How can you improve your method?

Second approximation

Generalization:



Riemann definition of Definite Integral: if f is a continuous function on [a,b] and this interval is equally divided into n intervals of width $\Delta x = \frac{b-a}{n}$, and if $x_i = a + i\Delta x$ is the right endpoint of subinterval i, then:

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)dx$$

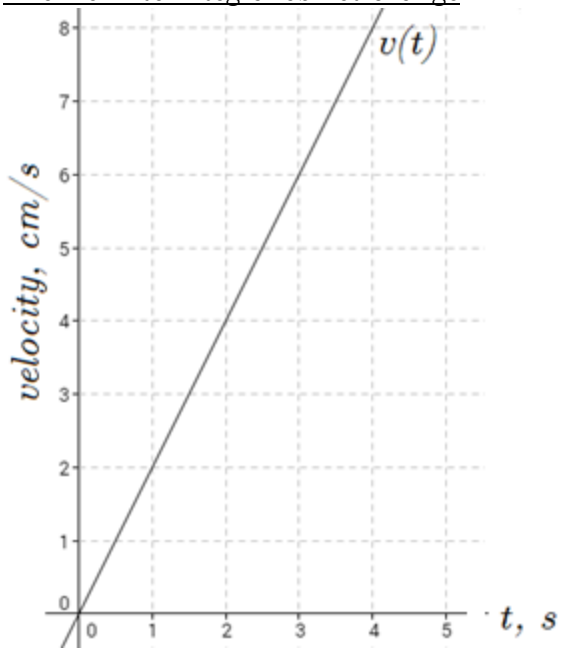
The Fundamental Theorem of Calculus: Part 2

If $f(x)$ is the derivative of $F(x)$, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

For real numbers a and b (called the limits of integration). It is not required that $a < b$.

The Definite Integral as net change



An object moves along the x-axis such that its velocity in cm/s is given by $(t) = 2t$. At time $t = 0\ s$, the object is at the origin. After 3 seconds, how far as the object traveled?

1. Find the specific position function $x(t)$.

2. Use the position function to find the difference between the positions ("displacement") at time $t = 3$ and time $t = 0$.

3. Find the exact area (using geometry) under the velocity function in the same time interval as problem 2. Use units in your calculations.

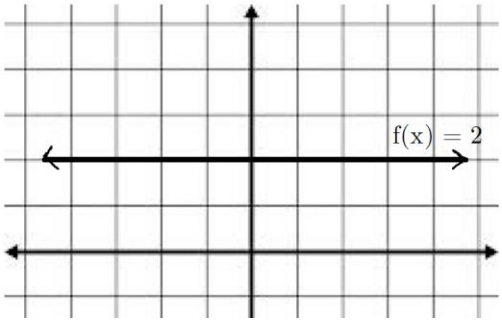
4. Write a *definite* integral that will find the displacement. Then use the second FTC to evaluate the integral.

5. In a complete sentence, write a conjecture about what you think the definite integral can be used to find.

Connection between Area and Antiderivatives and Slope

For each function, use geometry to find the area A(x) *under the function* f(x) *between* -1 and some arbitrary point x (or, over the interval [-1,x]). *Then, find* A'(x). What do you notice about f(x) and A'(x)?

1.

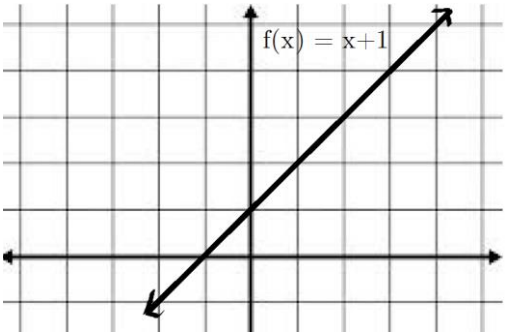


f(x) = 2

Area function A(x) =

A'(x) or $\frac{dA}{dx}$ =

2.

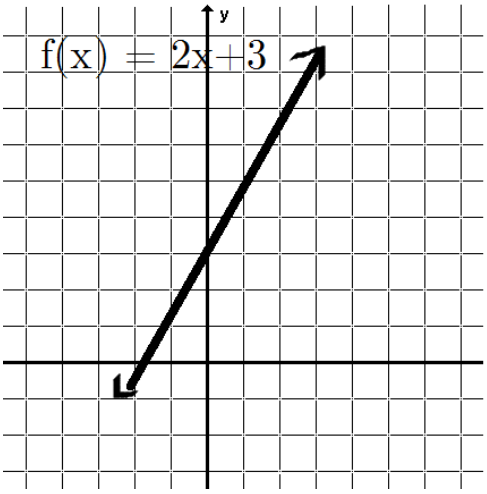


f x) = x+1

Area function A(x) =

A'(x) or $\frac{dA}{dx}$ =

3.



f (x) = 2x+3

Area function A(x) =

A'(x) or $\frac{dA}{dx}$ =

Now go back and find the area under the curve using the FTC:

1. $\int_{-1}^{x} 2\,dx$

2. $\int_{-1}^{x} x + 1\,dx$

3. $\int_{-1}^{x} 2x + 3\,dx$

AP Calculus AB Name_____

Definite Integrals Practice

Date_____

Evaluate each definite integral.

1) $\int_{1}^{4} -\frac{1}{x^3}\,dx$

2) $\int_{1}^{4} (-x + 2)\,dx$

3) $\int_{0}^{3} (-2x - 1)\,dx$

4) $\int_{-3}^{1} (-2x - 2)\,dx$

5) $\int_{1}^{4} -\frac{2}{x}\,dx$

6) $\int_{1}^{3} (x^3 - 4x^2 + 4)\,dx$

7) $\int_{-3}^{-1} (2x^2 + 12x + 14)\,dx$

8) $\int_{-2}^{-1} \frac{2}{x^3}\,dx$

9) $\int_{1}^{4} (x - 1)\,dx$

10) $\int_{-4}^{-1} -\frac{4}{x}\,dx$