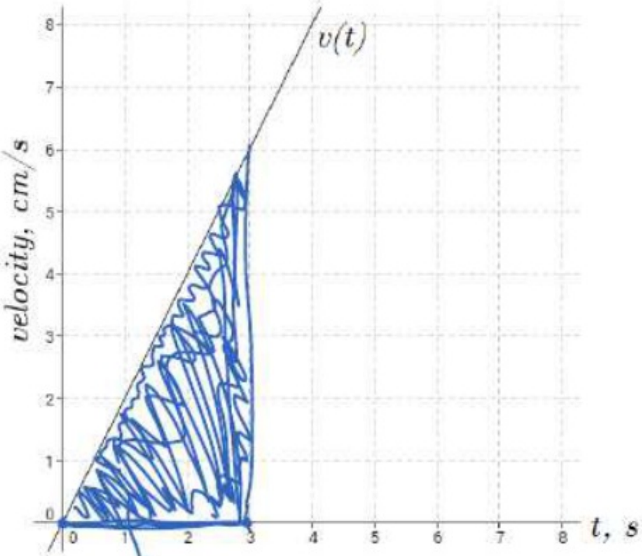
The Fundamental Theorem of Calculus: Part 2 (Part 1 is harder!)

If $f(x)$ is the derivative of $F(x)$, then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

For real numbers a and b (called the limits of integration). It is not required that $a < b$.
Note: not yet proven or intuitively understood, just didn't want to obscure the easy stuff!

The Definite Integral as net change
An object moves along the x-axis such that its velocity in cm/s is given by $(t) = 2t$. At time $t = 0\ s$, the object is at the origin. After 3 seconds, how far as the object traveled?



1. Find the specific position function $x(t)$.

$x(t) = t^2 + C$ gen
$x(0) = 0 = 0^2 + C$
$\underline{0 = C}$
$x(t) = t^2$

2. Use the position function to find the difference between the positions ("displacement") at time $t = 3$ and time $t = 0$.

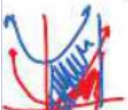
$x(3) = 3^2 = 9$
$x(0) = 0^2 = 0$
$9 - 0 = 9_{cm}$

3. Find the exact area under the velocity function in the same time interval as problem 2. Use units in your calculations.

$A = \frac{1}{2} \cdot b \cdot h = \frac{1}{2}(3\ sec)(6\ cm/sec) = \boxed{9\ cm}$
$A = \frac{1}{2}(t)(v(t))$

$\cancel{sec} \cdot \frac{cm}{\cancel{sec}}$

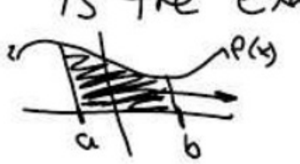
4. Write a *definite* integral that will find the displacement. Then use the second FTC to evaluate the integral.



$\int_0^3 v(t)dt = \int_0^3 2t\,dt = \underbrace{t^2 + C}_{F(t)}\Big|_0^3 = (3^2 + C) - (0^2 + C)$
not nec.
$F(3) - F(0)$

5. In a complete sentence, write a conjecture about what you think the definite integral can be used to find.

$\int_a^b f(x)dx$ is.. the displacement of $F(b)$; $F(a)$.
$9 + C - C = 9$
is the exact area under the curve. from a to b



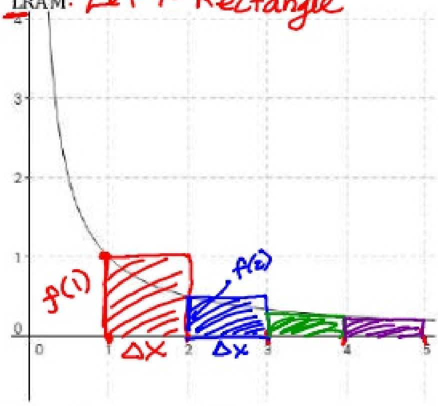
But will this assumption, that the definite integral will compute area under a curve, work for non-linear functions? Until we prove this conjecture, we will have to estimate area using traditional methods. This will lead us to the definition of a definite integral as the infinite limit of Riemann sums.

What is a Riemann sum? A Riemann sum is a method of approximating areas under curves by using basic geometry to partition the coordinate plane.

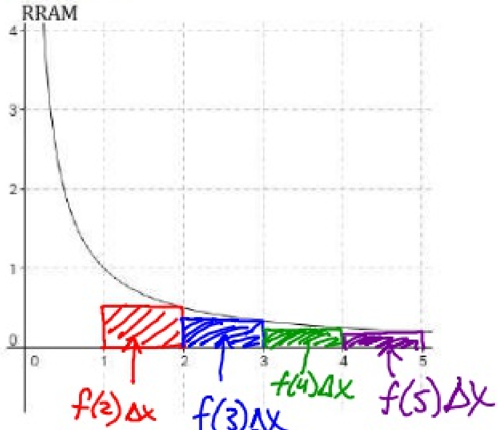
There are four basic types of Riemann sums to be familiar with for the AP test. They are abbreviated as LRAM, RRAM, MRAM, and TRAM. The AM stands for Approximation Method and the other letters are for Left Rectangle, Right Rectangle, Midpoint Rectangle, and TRapezoid.

Let's try it:
Use LRAM, RRAM, and MRAM to approximate the area under $f(x) = \frac{1}{x}$ on the interval [1,5] using n=4 subintervals.

LRAM: Left-Rectangle



RRAM



$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = \frac{4}{4} = 1$ (width)

$\Delta x\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right) = 1(1.28\overline{3})$

$\boxed{\approx 1.283}$

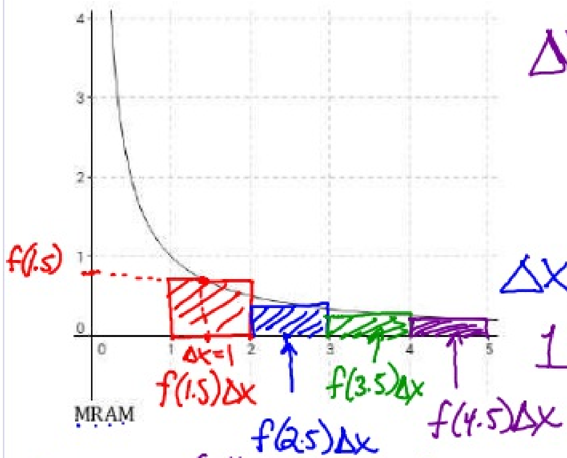
$f(1)\cdot\Delta x + f(2)\cdot\Delta x + f(3)\Delta x + f(4)\Delta x$

$\Delta x\left(f(1)+f(2)+f(3)+f(4)\right)$

$\sum_{i=1}^{4} f(i)\Delta x$

$\Delta x\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$

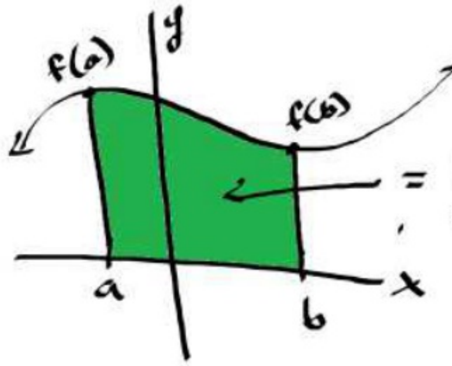
$1\left(\frac{25}{12}\right) = \frac{25}{12} \approx 2.083$



MRAM

$A \approx \Delta x\left(\sum_{i=1}^{4} f(i+0.5)\right)$

$A \approx 1.575$



$y = f(x)$

$$= \int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a).$$

For n-subintervals

$$\text{Area under curve} \approx \underbrace{\Delta x}_{\text{width of Rect.}} \cdot \underbrace{\left(f(x_1)+f(x_2)+\cdots+f(x_n)\right)}_{\text{height of Rect.}}$$

$$\text{Area} \approx \Delta x \cdot \sum_{i=1}^{n} f(x_i)$$

$$\text{Area} = \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(x_i)$$

$$\Delta x = \frac{b-a}{n}$$

$$\lim_{n\to\infty} \frac{b-a}{n} \cdot \sum_{i=1}^{n} f(x_i)$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) \cdot dx$$

infinitely small Δx

(Def'n of Definite Integral)

966) $\frac{1}{2}\int_{-1}^{1} 2x(x^2+1)^3\,dx$

$\frac{1}{2}\,\frac{1}{4}(x^2+1)^4$

$\left.\frac{1}{8}(x^2+1)^4\right]_{-1}^{1}$

$\frac{1}{8}(1^2+1)^4 - \frac{1}{8}((-1)^2+1)^4$

$\frac{1}{8}(2)^4 - \frac{1}{8}(2)^4 = 0$

Due Friday (#967 - 972, #1004 - 1007)