D-AD2 Practice Assessment SOLUTIONS

For each function below, find the derivative function.

1. $f(x) = 4\sqrt[3]{x^2} + 2x - \frac{1}{x}$

   Rewrite: $f(x) = 4x^{\frac{2}{3}} + 2x - x^{-1}$

   Power rule: $f'(x) = 4 * \frac{2}{3}x^{-\frac{1}{3}} + 2 - -1x^{-2}$ → *Simplify*: $f'(x) = \frac{8}{3}x^{-\frac{1}{3}} + 2 + \frac{1}{x^2}$ → $\boxed{f'(x) = \frac{8}{3x^{1/3}} + 2 + \frac{1}{x^2}}$

2. $g(t) = -2\cos t$

   $g'(t) = -2 * -\sin t$ → $\boxed{g'(t) = 2\sin t}$

3. $y = 5^x + \csc x - \tan x$

   $\boxed{y' = 5^x * \ln 5 - \csc x \cot x - \sec^2 x}$ *Just do it rule by rule*

4. $s(t) = e^{3t}$

   $s'(t) = e^{3t} * 3$ → $\boxed{s'(t) = 3e^{3t}}$

   (exponential derivative + chain rule)

D-AD2b

5. ~~Find $\frac{dy}{dx}$ if $y = \sec^{-1} x$~~ ~~Not on 10/26 test~~

   $\cancel{\boxed{\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}}}$ ~~*Directly from formula booklet*~~

See next page for this year's #5

6. If $y = \ln(5x + 1)$, find $\frac{dy}{dx}\big|_{x=1}$

   Note that this problem has you find the numerical slope value, so plug x=1 in after finding dy/dx.

   $\frac{dy}{dx} = \frac{1}{5x+1} * 5$ → $\frac{dy}{dx} = \frac{5}{5x+1}$ Now plug x=1 in: $\boxed{\frac{dy}{dx} = \frac{5}{5(1)+1} = \frac{5}{6}}$

   (natural log rule and chain rule)

7. Find the derivative of $y = \tan(3x^2 - 3)$

   $y' = \sec^2(3x^2 - 3) * 6x$ → $\boxed{y' = 6x\sec^2(3x^2 - 3)}$

   (trig derivative and chain rule)

D-AD3

Let f(x)=$5x^2 - 3x + 5$ and g(x)=$x^2 + \cos x$

8. If $h(x) = f(x)g(x)$, find $h'(x)$. [No need to simplify.]

   $h(x) = (5x^2 - 3x + 5)(x^2 + \cos x)$

   List out ingredients: $f: 5x^2 - 3x + 5$ $\quad g: x^2 + \cos x$

   $f' = 10x - 3$ $\quad g': 2x - \sin x$

   Product Rule recipe: $f'g + fg'$

   $\boxed{h'(x) = (10x - 3)(x^2 + \cos x) + (5x^2 - 3x + 5)(2x - \sin x)}$

9. If $p(x) = \frac{f(x)}{g(x)}$, find $p'(x)$ [No need to simplify]

   List out ingredients: $f: 5x^2 - 3x + 5$ $\quad g: x^2 + \cos x$

   $f' = 10x - 3$ $\quad g': 2x - \sin x$

   Quotient Rule recipe: $\frac{f'g - fg'}{g^2}$ $\boxed{\frac{(10x-3)(x^2+\cos x)-(5x^2-3x+5)(2x-\sin x)}{(x^2+\cos x)^2}}$

D-AD2 Practice Assessment

For each function below, find the derivative function.

1. $f(x) = 4\sqrt[3]{x^2} + 2x - \frac{1}{x}$

2. $g(t) = -2\cos t$

3. $y = 5^x + \csc x - \tan x$

4. $s(t) = e^{3t}$

D-AD2b

5. Find $\frac{dy}{dx}$ if $y = (5x^2 + 3)^{80}$

$80\underbrace{(5x^2+3)^{79}}_{\text{chain}} \cdot 10x \Rightarrow \boxed{800x(5x^2+3)^{79}}$

6. If $y = \ln(5x + 1)$, find $\frac{dy}{dx}|_{x=1}$

7. Find the derivative of $y = \tan(3x^2 - 3)$

D-AD3

Let $f(x) = 5x^2 - 3x + 5$ and $g(x) = x^2 + cos\, x$

8. If $h(x) = f(x)g(x)$, find $h'(x)$. [No need to simplify.]

9. If $p(x) = \frac{f(x)}{g(x)}$, find $p'(x)$ [No need to simplify]

D-AD4

10. Calculate the derivative of y=$\sqrt[3]{6x^2-3x+1}$

Rewrite: $y=(6x^2-3x+1)^{1/3}$

Chain rule: $y'=\frac{1}{3}(6x^2-3x+1)^{-\frac{2}{3}}*(12x-3)$

Simplify: $y'=\frac{12x-3}{3(6x^2-3x+1)^{\frac{2}{3}}}$ → $\boxed{y'=\frac{4x-1}{(6x^2-3x+1)^{\frac{2}{3}}}}$

11. If y=$\cos^2(3x-12)$, find $\frac{dy}{dx}$.

Rewrite: $y=[\cos(3x-12)]^2$

Chain rule....twice!! : $\frac{dy}{dx}=2\,[\cos(3x-12)]^1*-\sin(3x-12)*3$ → $\boxed{-6\sin(3x-12)\cos(3x-12)}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	3	-2
2	3	$\frac{3}{2}$	1	$-\frac{1}{2}$
3	4	$-\frac{1}{2}$	2	$\frac{3}{2}$
4	2	-2	4	2

12. Use the table to find $h'(1)$ if $h(x)=f(g(x))$.

Chain rule by definition:

$$h'(x)=f'(g(x))*g'(x)$$
$$h'(1)=f'(g(1))*g'(1)$$
$$h'(1)=f'(3)*-2$$
$$h'(1)=-\frac{1}{2}*-2\rightarrow\boxed{1}$$

D-CD4

13. Show that $f(x)=\begin{cases}5x^2-3x-6 & x\le 1\\ -2x^2-2 & x>1\end{cases}$ is not differentiable at x=1.

Check continuity at x=1

From left and at 1: $f(1^-)=5(1)^2-3(1)-6\ \rightarrow -4$

From right $f(1^+)=-2(1^2)-2\rightarrow -2-2\rightarrow -4$ $-4=-4$, so yes continuous

Check differentiability at x=1

$f'(x)=\begin{cases}10x-3 & x\le 1\\ -4x & x>1\end{cases}$

Slope from left: $f'(1^-)=10(1)-3=7$

Slope from right: $f(1^+)=-4(1)=-4$ $7\ne -4$ so, not differentiable at x=1.

14. Find the values of a and b that would make f(x) differentiable. $f(x)=\begin{cases}ax^2+bx-2 & x\le 2\\ -2x^2+2x+8 & x>2\end{cases}$

Continuous? Check 2 from left, right, and exactly 2

Left and middle = Right

$a(2^2)+b(2)-2=-2(2^2)+2(2)+8$

$4a+2b-2=\ -8+4+8$

$\mathbf{4a+2b=\ 6}$

Differentiable? Check 2 from left, right, and exactly 2 of $f'(x)$

$f'(x)=\begin{cases}2ax+b & x\le 2\\ -4x+2 & x>2\end{cases}$

Left and middle = Right

$2a(2)+b\ =\ -4*2+2$

$\mathbf{4a+b\ =\ -6}$

$-\begin{cases}\mathbf{4a+2b=6}\\ \mathbf{4a+b=-6}\end{cases}$ Solve system of equations

$b=12$

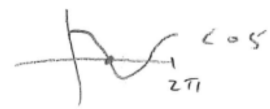
$4a+12=-6$

$4a\ =-18$

$a\ =-\frac{18}{4}$

D-CD7

15. Write the equation of a line tangent to $y = 3\cos(x) + 2$ where $\boxed{x = \frac{\pi}{2}}$



$y - y_1 = m(x - x_1)$

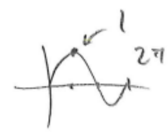
$y(\frac{\pi}{2}) \qquad y'(\pi/2) \qquad \frac{\pi}{2}$

$\boxed{y - 2 = -3(x - \frac{\pi}{2})}$

$\boxed{y_1}$ $y(\frac{\pi}{2}) = 3\cos(\frac{\pi}{2}) + 2$

$3(0) + 2 = \boxed{2}$

$\boxed{m}$



$y = 3\cos(x) + 2$

$y' = -3\sin(x) + 0 \Rightarrow -3\sin(x)$

$y'(\frac{\pi}{2}) = -3\sin(\frac{\pi}{2})$

$= -3(1) = \boxed{-3}$