

D-AD2

1. If $f(x) = 2\sqrt[3]{x^2}$, find $f'(2)$

$$f(x) = 2x^{\frac{2}{3}} \quad \text{Rewrite function using identity that } x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

$$f'(x) = 2 * \frac{2}{3}x^{-\frac{1}{3}} \quad \text{Power Rule}$$

$$f'(x) = \frac{4}{3} * \frac{1}{x^{\frac{1}{3}}}$$

$$f'(x) = \frac{4}{3\sqrt[3]{x}} \quad \text{Simplify}$$

$$f'(2) = \boxed{\frac{4}{3\sqrt[3]{2}}} \quad \text{Plug in 2 for x...can't simplify (note that, if x were 8, you could!)}$$

2. If $g(\theta) = -5 \cot \theta$, find $g'(\frac{\pi}{2})$ Note typo

$$g'(\theta) = -5 * -\csc^2 \theta \quad \text{Trig derivative rule}$$

$$g'(\theta) = 5 \csc^2 \theta$$

$$g'(\frac{\pi}{2}) = 5 \csc^2 \frac{\pi}{2}$$

$$g'(\frac{\pi}{2}) = 5 * \left(\frac{1}{\sin \frac{\pi}{2}} \right)^2 \quad \text{Cosecant is } 1/\sin$$

$$g'(\frac{\pi}{2}) = 5 * \left(\frac{1}{1} \right)^2 \quad \sin(\frac{\pi}{2}) = 1$$

$$g'(\frac{\pi}{2}) = 5 * 1 = \boxed{5}$$

3. $y = e^{3x} - \cos x - \sec x$. Find the slope of y when $x=0$.

Note change and typo

$$y' = e^{3x} * 3 - -\sin x - \sec x \tan x$$

chain rule trig trig

$$y' = 3e^{3x} + \sin x - \sec x \tan x$$

$$y'(0) = 3e^{3*0} + \sin 0 - \sec 0 \tan 0$$

$$= 3e^0 + 0 - 1 * 0$$

$$= 3(1) + 0 - 0 \rightarrow \boxed{3}$$

4. $s(t) = 3^t$ Find $s'(0)$

$$s'(t) = 3^t \ln 3 \quad \text{Exponential rule}$$

$$s'(0) = 3^0 \ln 3$$

$$s'(0) = 1 * \ln 3 \rightarrow \boxed{\ln 3}$$

D-AD2b

5. Find $f'(2)$ if $f(x) = \ln 3x^2 + 5x$

$$\begin{aligned} f'(x) &= \frac{1}{3x^2} \cancel{6x} + 5 && \text{Ln rule + chain rule, slope of } 5x \text{ is 5} \\ f'(x) &= \frac{6x}{3x^2} + 5 \\ f'(x) &= \frac{2}{x} + 5 && \text{Simplify} \\ f'(2) &= \frac{2}{2} + 5 \rightarrow \frac{1}{5} = 6 \end{aligned}$$

6. Find the slope of $y = \sqrt[3]{2x}$ where $x=0$.

$$\begin{aligned} y &= (2x)^{\frac{1}{3}} && \text{Rewrite} \\ y' &= \frac{1}{3}(2x)^{-\frac{2}{3}} * 2 && \text{Power rule + chain rule} \\ y' &= \frac{1}{3(2x)^{2/3}} * 2 \\ y' &= \frac{2}{3(2x)^{2/3}} \\ y'(0) &= \frac{2}{3(2*0)^{\frac{2}{3}}} \rightarrow \frac{2}{3*0} \rightarrow \frac{2}{0} && (\text{undefined slope!}) \text{ so a vertical tangent here.} \end{aligned}$$

7. Find the derivative of $y = \cos^{-1} 3x$

$$\begin{aligned} y &= \arccos 3x \\ y' &= -\frac{1}{\sqrt{1-(3x)^2}} * 3 && \text{May help to rewrite this way if you want} \\ y' &= -\frac{3}{\sqrt{1-9x^2}} && \text{Inverse trig rule + chain rule} \\ &&& \text{Simplify} \end{aligned}$$

8. Find the derivative of $y = \log_4 x^2$

$$\begin{aligned} y' &= \frac{1}{x^2 \ln 4} * \cancel{2x} && \text{Log derivative + chain rule} \\ y' &= \frac{2x}{x^2 \ln 4} \rightarrow \frac{2}{x \ln 4} && \text{Simplify} \end{aligned}$$

D-AD4

9. Find $y'(1)$ when $y = \sqrt[3]{5x+1}$

$$\begin{aligned} y &= (5x+1)^{\frac{1}{3}} && \text{Rewrite so power rule is applicable} \\ y' &= \frac{1}{3}(5x+1)^{-\frac{2}{3}} * 5 && \text{Power rule + chain rule} \\ y' &= \frac{1}{3} * \frac{1}{(5x+1)^{\frac{2}{3}}} * 5 && \text{Simplify negative exponent} \\ y' &= \frac{5}{3(5x+1)^{\frac{2}{3}}} && \text{Combine fractions} \\ y'(1) &= \frac{5}{3(5*1+1)^{\frac{2}{3}}} \rightarrow \frac{5}{3(6)^{\frac{2}{3}}} \rightarrow \frac{5}{3(6^{\frac{1}{3}})} \rightarrow \frac{5}{3^{\frac{3}{3}}\sqrt[3]{36}} && \text{Plug in 1, simplify using exponent rules} \end{aligned}$$

10. Find the derivative of $y = \tan^2 3x$

$$\begin{aligned} y &= [\tan 3x]^2 && \text{Rewrite} \\ y' &= 2[\tan 3x]^1 * \sec^2 3x * 3 && \text{Chain rule...twice!} \\ &&& \text{Simplify} \end{aligned}$$

11.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	2	1	1
2	4	$\frac{1}{2}$	2	$\frac{3}{2}$
3	3	$-\frac{3}{2}$	4	0
4	1	-2	2	-2

Part 1) Given $h_1(x) = (f(x))^2$, find $h_1'(2)$ Part 2) Given $h_2(x) = f(g(x))$, find $h_2'(2)$

Part 1: $h(x) = [f(x)]^2$

$$h'(x) = 2[f(x)]^1 f'(x)$$
 Power rule + Chain rule

$$h'(2) = 2[f(2)]^1 f'(2)$$
 Plug in 2 for x

$$h'(2) = 2[4] * \frac{1}{2}$$
 $f(2) = 4$ in table, and $f'(2) = \frac{1}{2}$

$$h'(2) = \boxed{4}$$

Part 2: $h(x) = f(g(x))$

$$h'(x) = f'(g(x)) * g'(x)$$
 Chain rule by definition

$$h'(2) = f'(g(2)) * g'(2)$$
 Plug in 2 for x

$$h'(2) = f'(2) * \frac{3}{2}$$
 $g(2) = 2$ from table and $g'(2) = \frac{3}{2}$

$$h'(2) = \frac{1}{2} * \frac{3}{2}$$
 $f'(2) = \frac{1}{2}$ from table

$$h'(2) = \boxed{\frac{3}{4}}$$