

Practice Assessment

1. The position, in feet, of a particle moving along a straight path is given by the differentiable function  $x(t) = \sin 2t - \cos 4t$  where  $t$  is measured in seconds. Find the acceleration of the particle at  $t=0$ .  
Include units in your answer.

$$x(t) = \sin 2t - \cos 4t$$

$$x'(t) = v(t) = \cancel{\cos 2t \cdot 2} - \cancel{-\sin 4t \cdot 4} \\ = 2 \cos 2t + 4 \sin 4t$$

$\begin{matrix} \text{pos} \rightarrow v \rightarrow \text{acc} \\ f \quad f' \quad f'' \end{matrix}$

$$x''(t) = v'(t) = a(t) = -2 \sin 2t \cdot 2 + 4 \cos 4t \cdot 4 \\ a(t) = -4 \sin 2t + 16 \cos 4t$$

$$a(0) = -4 \sin(2 \cdot 0) + \frac{16 \cos(4 \cdot 0)}{2} \\ = 16 \text{ ft/s}^2$$

2. The position, in feet, of a particle moving along a straight path is given by the differentiable function  $s(t) = -t^3 + 5t^2 - 7t + 3$ . Find all times  $t$  where the particle is at rest. means  $\underline{\text{velocity}} = 0$

Let  $x = t$

$$s = -x^3 + 5x^2 - 7x + 3$$

$$\frac{ds}{dt} = -3x^2 + 10x - 7 = 0 \quad \text{"at rest"}$$

$$-1(3x^2 - 10x + 7) = 0$$

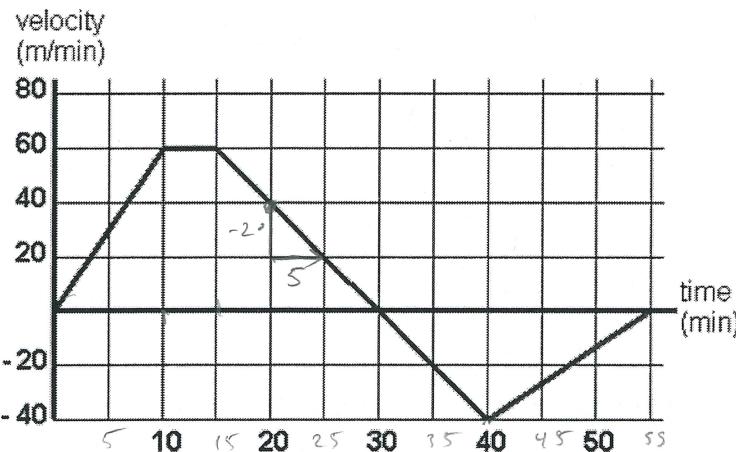
$$-1(3x - 7)(x - 1) = 0$$

$$x = \frac{7}{3} \quad x = 1$$

$$t = \frac{7}{3} \text{ sec}$$

$$t = 1$$

3. A person is hiking in a national park and their velocity in meters per minute is graphed below.



When does the person change direction?  
"Velocity changes sign"

$$t = 30 \text{ min}$$

When is the person walking at her greatest speed?

$$\boxed{v(t)}$$

$t = (10, 15) \text{ or, from } 10 \text{ min to } 15 \text{ min.}$

When is the person slowing down?

" $v(t)$  approaches 0"

$(15, 30)$  interval and  $(40, 55)$

Calculate the acceleration at  $t=20 \text{ min.}$

$$\boxed{v'(t)}$$

Slope of velocity

$$a(t) = v'(t) = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{-20 \text{ m/min}}{5 \text{ min}} = -4 \text{ m/min}^2$$

## D-AD5

4. Find the slope of the tangent line at  $x = -1$  when  $y = xy + x^2 + 1$

① Point? Plug  $x = -1$  to find  $y$ .  $y = -1y + (-1)^2 + 1 \rightarrow (-1, 1)$

② Slope?  $\frac{dy}{dx} [y] = [xy + x^2 + 1] \frac{1}{dx}$

$$\frac{dy}{dx} \equiv 1 \cdot y + x \cdot \frac{dy}{dx} + 2x + 0$$

product rule

$$2y = 2 \rightarrow y = 1$$

factor  $\frac{dy}{dx} (1-x) = y + 2x$

$$\frac{dy}{dx} = \frac{y + 2x}{1-x}$$

③ plug in  $(-1, 1)$

$$\frac{1+2(-1)}{1-(-1)} = \frac{-1}{2} = \left(\frac{-1}{2}\right)$$

5. If  $x^2 + xy + y^3 = 0$ , then find  $\frac{dy}{dx}$

$$\frac{1}{dx} [x^2 + xy + y^3 = 0] \frac{1}{dy}$$

$$2x + 1 \cdot y + x \cdot \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$

chain rule

$$x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -2x - 1y$$

$$\frac{dy}{dx}(x + 3y^2) = -2x - 1y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - 1y}{x + 3y^2}}$$

## D-CD7

6. Write the equation of the line tangent to  $y = (2x-1)^4$  where  $x=1$

① point:  $y = (2(1)-1)^4 \rightarrow (2-1)^4 = 1$   $(1, 1)$

② Slope:  $\frac{dy}{dx} = 4(2x-1)^3 \cdot 2 \rightarrow 8(2x-1)^3$

plug in  $x=1$

$$8(1)^3 \rightarrow 8$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 1 = 8(x - 1)}$$

$$\text{or } \boxed{y = 8x - 7}$$

7. Write the equation of a line with a slope of 6 that is tangent to  $y = x^2 - 4x + 3$ .

Given Slope!  $\frac{dy}{dx} = 6$   $\frac{dy}{dx} = 2x - 4 = 6$   
Need point.

$$2x = 10$$

$$\underline{x = 5}$$

$$y = x^2 - 4x + 3$$

$$= 5^2 - 4 \cdot 5 + 3$$

$$= 25 - 20 + 3$$

$$= 8$$

$$(5, 8) \rightarrow \boxed{y - 8 = 6(x - 5)}$$