PRACTICE

Mr. [illegible] 10-14-15

D-CD3

1. Use the limit definition of the derivative to show that for $f(x) = 3x^2 + 2x - 1$, $f'(x) = 6x + 2$ Show all steps/work.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow \lim_{h\to 0} \frac{3(x+h)^2 + 2(x+h) - 1 - (3x^2 + 2x - 1)}{h}$$

$$\lim_{h\to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h - 1 - 3x^2 - 2x + 1}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + h^2 + \cancel{2x} + 2h \cancel{- 1} \cancel{- 3x^2} \cancel{- 2x} \cancel{+ 1}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(6x + h + 2)}{\cancel{h}}$$

$$\lim_{h\to 0} 6x + h + 2 = 6x + 2$$

2. Find the slope of a line tangent to the curve $f(x) = 3x^2 + 2x - 1$ at the point (1,4).

DERIVATIVE

x $\quad f(x)$

$f'(x) = 6x + 2$

$f'(1) = 6(1) + 2 = 6 + 2 = 8$

D-CD1

3. $g(x) = \pi^3$. Find $g'(x)$

$\pi^3 \leftarrow$ a constant $\quad g'(x) = 0$

rule: $\frac{d}{dx}[c] = 0$

4. $y = \sqrt{x}$ . Find $\frac{dy}{dx}$

$y = x^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{1}{2x^{1/2}} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{2\sqrt{x}}}$

5. $f(x) = \underbrace{4x^3}_{f}\underbrace{\sin(x)}_{g}$. Find $f'(x)$

$f = 4x^3 \qquad g = \sin(x)$

$f' = 12x^2 \qquad g' = \cos(x)$

PRODUCT RULE: $f'g + fg'$

$(12x^2 \cdot \sin(x)) + 4x^3 \cdot \cos(x)$

$f'(x) \quad 12x^2\sin(x) + 4x^3\cos(x)$

6. $y = \frac{2x}{3x-1}$ . Find y'

$f = 2x \qquad g = 3x - 1$

$f' = 2 \qquad g' = 3$

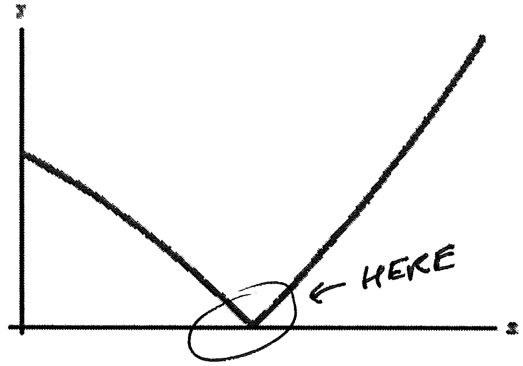
QUOTIENT RULE: $\frac{f'g - fg'}{g^2}$

$$y' = \frac{2(3x-1) - 2x \cdot 3}{(3x-1)^2}$$

$$y' = \frac{\cancel{6x} - 2 \cancel{- 6x}}{(3x-1)^2} \Rightarrow y' = \frac{-2}{(3x-1)^2}$$

D-CD4

7. Indicate on the graph below where f(x) has a non-differentiable point. Explain in words why.

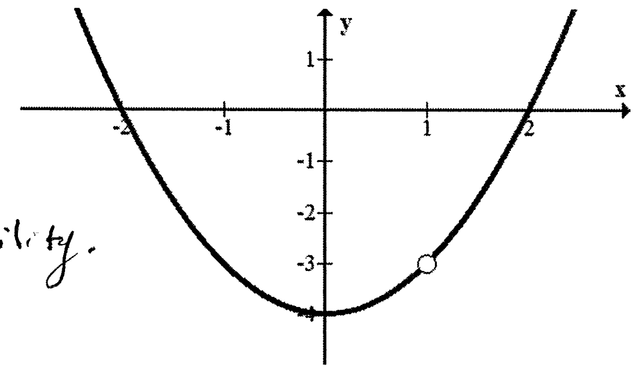



Slope changes from negative to positive without passing thru 0 (flat slope).

There is not a unique tangent line with a defined slope.

Also Mr. Pickle car gets stuck.

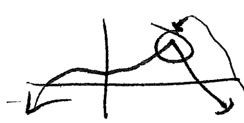
8. Is the function shown differentiable on the interval (-2,2)? Explain.




No. It is not continuous on (-2, 2) and continuity is a necessary condition for differentiability.

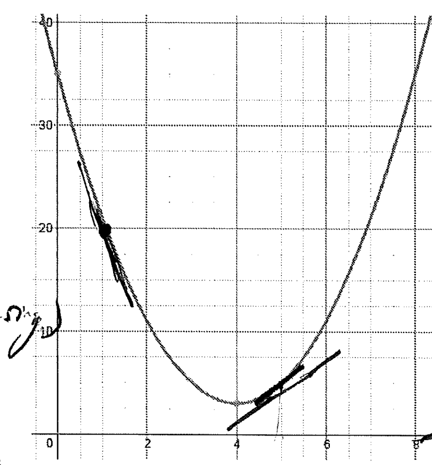
9. True or false: All differentiable functions are continuous, but not all continuous functions are differentiable.

True




D-CD2

Suppose the volume of water (in liters) in a holding tank after $t$ minutes is modeled by the function $V(t) = 2t^2 - 16t + 35$. A sketch of the graph is shown.




10. Is the volume of water increasing or decreasing at t = 1 minute? Explain using derivatives.

Decreasing. $v'(t) = 4t - 16$ liters/min

$v'(1) = 4(1) - 16 = -12$ l/min (negative, so decreasing)

11. Is the volume of water increasing or decreasing at t = 5 minutes? Explain using derivatives.

Increasing. $v'(t) = 4t - 16$ liters/min

$v'(5) = 4(5) - 16 = 4$ liters/min (positive, so increasing)

12. Is the volume of water changing faster at t = 1 or t = 5? Explain using derivatives.

At t=1.

$-12$ is a larger absolute value than $4$, so the decrease is faster.

$v'(1)$ $v'(5)$

Also, slope at t=1 is steeper than at t=5.