Good afternoon and welcome back! Warm up in notes:

$y - y_1 = m(x - x_1)$

Find the equation of the line tangent to $y=(x^2-2x-3)(2x+5)$ when $x=1$.

y_1?

plug 1 into the given function

$y(1) = (1^2 - 2 - 3)(2(1) + 5)$

$(-4)(7) = -28$

m?

take derivative, plug 1 into it

$\frac{dy}{dx} = (2x-2)(2x+5) + (x^2-2x-3)(2)$

@ $x=1$ $\quad (2(1)-2)(2(1)+5) + (-4)(2)$

$0 - 8 = -8$

$\boxed{y+28 = -8(x-1)}$

VRG

What can you take derivatives of?

$y = 3x \quad y' = ?$ Linear

$y = 3x^{90} - 4x + \sqrt[3]{x^7}$ Polynomial

$y = \sin \longrightarrow y' = \cos$

$y = \cos \longrightarrow y' = -\sin$

$y = \cot(x) \longrightarrow y' = -\csc^2(x)$

$y = \tan(x) \longrightarrow y' = \sec^2(x)$

trig

$y = \frac{3x+5}{x-2}$ Quotient Rule

Find the derivative of $y = \sec(x)$

Recall: $\sec(x) = \dfrac{1}{\cos(x)}$

SOHCAHTOA

$$y = \sec(x) = \frac{1}{\cos(x)} \begin{matrix} f \\ g \end{matrix}$$

quotient rule

$$y' = \frac{f'g - fg'}{g^2}$$

$f' = 0$

$g' = -\sin(x)$

$$\frac{dy}{dx} = \frac{\cancel{0 \cdot \cos(x)} - 1 \cdot -\sin(x)}{\cos^2(x)}$$

$$\frac{dy}{dx} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}$$

$\tan(x) \cdot \sec(x)$

$$\boxed{\sec(x) \cdot \tan(x)}$$

The six major trig derivative rules

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x)$$

$\sec^2$

$$\frac{d}{dx}\cot(x)$$

$-\csc^2(x)$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

How do you take the derivative of $y=\sin(3x^2-2)$?

The Chain Rule (one of the most important in calculus!)

How do you take the derivative of $y=\sin(3x^2-2)$?

Is it.....$dy/dx = \cos(3x^2-2)$?? $\cos(6x)$?

Let's look at it graphically.
bit.ly/chainrule02

No, both don't work.

[redacted]

So, no, the derivative is not so simple.

First, note that $y=\sin(3x^2-2)$ is a *composite function*

REVIEW

Suppose $f(x) = x^2$ and $g(x)=5x-3$

1. Find $f(g(x))$ (don't simplify)

$(5x-3)^2$

2. Simplify $f(g(x))$

$25x^2-30x+9$

3. Find $\frac{d}{dx} f(g(x))$

$50x-30$

$f \circ g$

$f(g(x))$

$h(x)=x^3$

$g(x)=3x-4$

$h(g(x))=(3x-4)^3$

So, if $f=x^2$ and $g=5x-3$

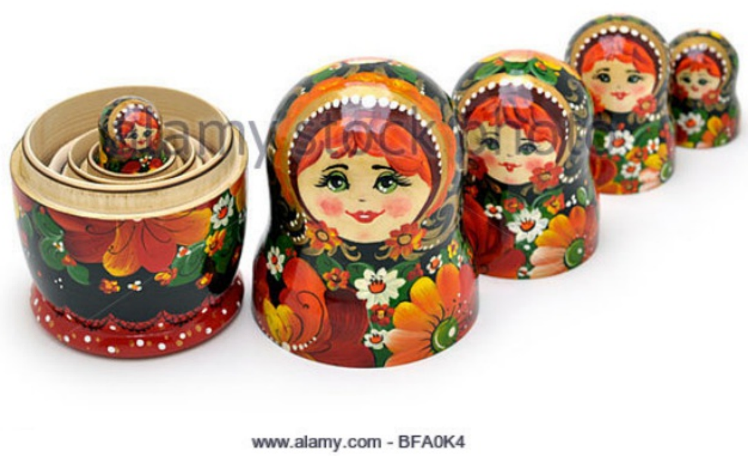
$f(g(x)) = (5x-3)^2 = 25x^2-30x+9$

Derivative of $f(g(x))$ would be $50x - 30$

$$10(5x-3)$$

$$2 \cdot 5(5x-3)$$

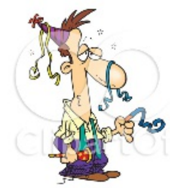
$$2(5x-3)' \cdot 5$$



The Chain Rule

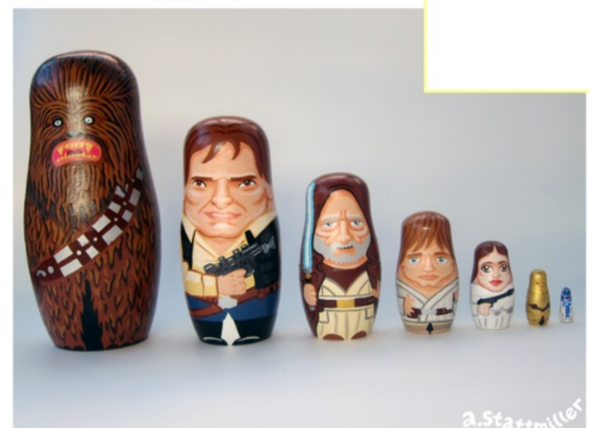
(booklet and notes)

$$\frac{d}{dx}\left[f(g(x))\right] = f\,'\,(g(x)\,) * g'(x)$$



$$\frac{d}{dx} f(g(x))$$

$$f'(g(x))\cdot g'(x)$$



Find f '(x) for f(x) = $(5x-3)^2$

$$2(5x-3)^1 \cdot 5$$

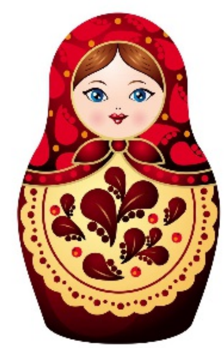
$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

Find f '(x) for f(x)=$(2x-3)^{100}$

$$f'(2x-3)$$

$$100(2x-3)^{99} \cdot 2$$

$$\boxed{200(2x-3)^{99}}$$

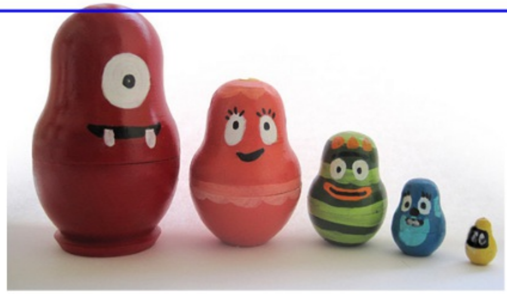


Find dy/dx where y = sec(3x^2+2)

$$y' = \sec(3x^2+2)\tan(3x^2+2)\cdot 6x$$

$$y = \sin(3x^2+2)$$

$$\frac{dy}{dx} = \cos(3x^2+2)\cdot 6x$$

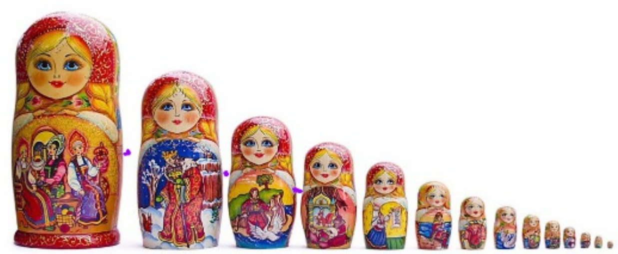


Find the derivative function

$y = [\cos(4x-3)]^{50} = \cos^{50}(4x-3)$

$\frac{dy}{dx} = 50[\cos(4x-3)]^{49} \cdot - \sin(4x-3) \cdot 4$

$-200[\cos(4x-3)]^{49} \sin(4x-3)$



24. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $\cancel{f}(0)$ is $f'(0)$

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$$\cancel{f'(x) = \frac{2}{3}(2x-2)^{-1/3}\cdot 2}$$

$$\cancel{f'(0) = \frac{2}{3}(-2)^{-1/3}\cdot 2}$$

$$\cancel{\frac{4}{3}(-2)^{-1/3}}$$

~~???~~

$$\frac{dy}{dx} = \frac{2}{3}(x^2 - 2x - 1)^{-1/3}\cdot(2x-2)$$

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{2}{3}(-1)^{-1/3}\cdot(-2)$$

$$\frac{2}{3}\left(\cancel{(-1)^{1/3}}\right)^{-1}(-2)$$

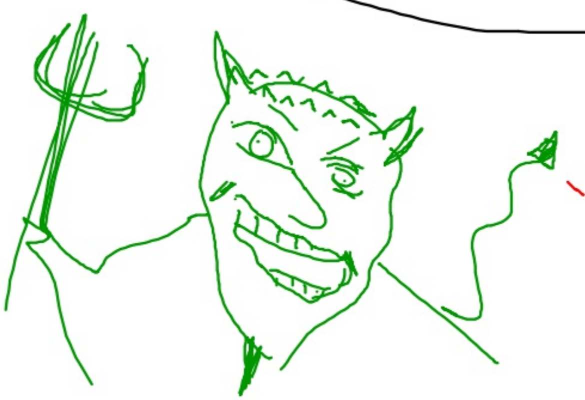
$$\sqrt[3]{-1}$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{2}{3}\left((-1)\right)^{-1}(-2)$$

$$\frac{1}{-1}$$

$$\frac{2}{3}(-1)(-2) = \frac{4}{3} \quad \text{A}$$



Time for practice in class!

(Use separate paper or Notebook)

Due Monday

2.8 The RULES: Power Product Quotient Chain

447. Let $f(x) = \begin{cases} 3-x & x<1 \\ ax^2+bx & x \geq 1 \end{cases}$ where a and b are constants.

a) If the function is continuous for all x, what is the relationship between a and b?

b) Find the unique values for a and b that will make f both continuous and differentiable.

448. Suppose that $u(x)$ and $v(x)$ are differentiable functions of x and that

$$u(1) = 2, \quad u'(1) = 0, \quad v(1) = 5, \quad \text{and} \quad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

a) $\frac{d}{dx}(uv)$ b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ d) $\frac{d}{dx}(7v-2u)$

449. Graph the function $y = \frac{4x}{x^2+1}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called *Newton's serpentine*.) Find the tangent lines at the origin and at the point $(1,2)$.

450. Graph the function $y = \frac{8}{x^2+4}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called the *witch of Agnesi*.) Find the tangent line at the point $(2,1)$.

FIND THE DERIVATIVE OF THE GIVEN FUNCTION. EXPRESS YOUR ANSWER IN SIMPLEST FACTORED FORM.

451. $A(z) = (3z-5)^4$

452. $q(u) = (3u^5 - 2u^3 - 3u - \frac{1}{3})^3$

453. $b(y) = (y^3-5)^{-4}$

454. $c(d) = \sqrt[3]{(5d^2-1)^5}$

455. $u(p) = \frac{3p^2-5}{p^3+2p-6}$

456. $V(x) = \frac{\sqrt{5x^3}}{5x^3}$

457. $f(x) = 3x^{1/3} - 5x^{-1/3}$

458. $g(z) = \frac{1}{\sqrt{36-z^2}}$

459. $p(t) = (3-2t)^{-1/2}$

460. $h(u) = \sqrt{u-1}\sqrt[3]{2u+3}$

461. $f(x) = \frac{3x}{x+5}$

462. $g(y) = \frac{4y-3}{3-2y}$

463. $p(x) = \frac{x^2+10x+25}{x^2-10x+25}$

464. $m(x) = \frac{7x}{1-3x}$

465. $f(x) = \frac{3}{x^2} - \frac{x^2}{3}$

466. $g(x) = \left(\frac{4x-3}{5-3x}\right)(2x+7)$

467. $F(x) = 10x^{27} - 25x^{1/5} + 12x^{-12} + 350$

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction. —*Leo Tolstoy*

Stay for DS tomorrow
mini lesson is important

Make a function Differentiable

$$f(x) = \begin{cases} 3-2x, & x \le 1 \\ ax^2+bx, & x>1 \end{cases} \longrightarrow f'(x) = \begin{cases} -2, & x \le 1 \\ 2ax+b, & x>1 \end{cases}$$

Continuity of f @ $x=1$

$$\lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$$

$$3-2(1) = 3-2(1) = a(1)^2+b(1)$$

$$\underline{\underline{1 = a+b}}$$

Differentiable/"Smooth" @ $x=1$? $\longrightarrow$ Cont. of f' @ $x=1$

$$\lim_{x\to 1^-} f'(1) = f'(1) = \lim_{x\to 1^+} f'(x)$$

$$-2 = -2 = 2a(1)+b$$

$$\underline{\underline{-2 = 2a+b}}$$

Solve the system of Es

(Elimination, Substitution, etc)

$$-\begin{cases} a+b=1 \\ (2a+b=-2) \end{cases}$$

$$-1a + \cancel{0}b = 3$$

$$-a = 3 \rightarrow a = -3$$

$$-3+b=1$$

$$b=4$$

$$\boxed{\begin{aligned} a &= -3 \\ b &= 4 \end{aligned}}$$