AP Calc Mini Lesson

Differentiability ("Smoothness")

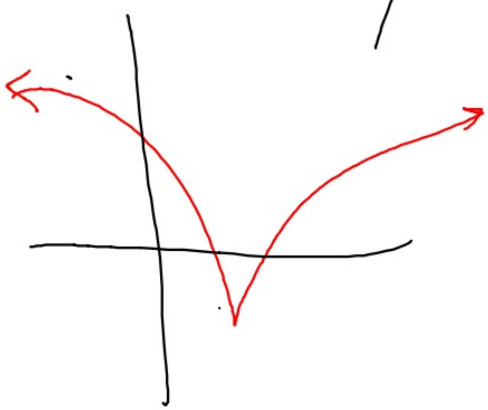
> A function f(x) is **<u>differentiable</u>** at a point if
> (a) f(x) is continuous at that point
> (b) f(x) has a *unique* tangent line with a *defined* slope

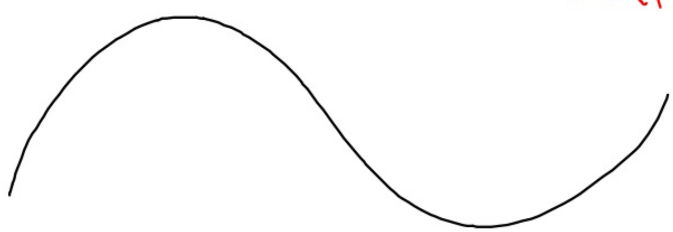
No ambiguity

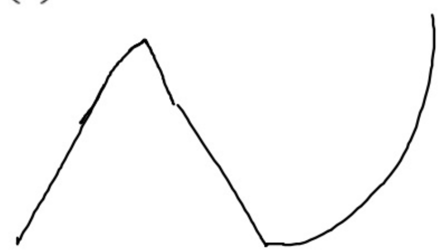
slope can't be vertical

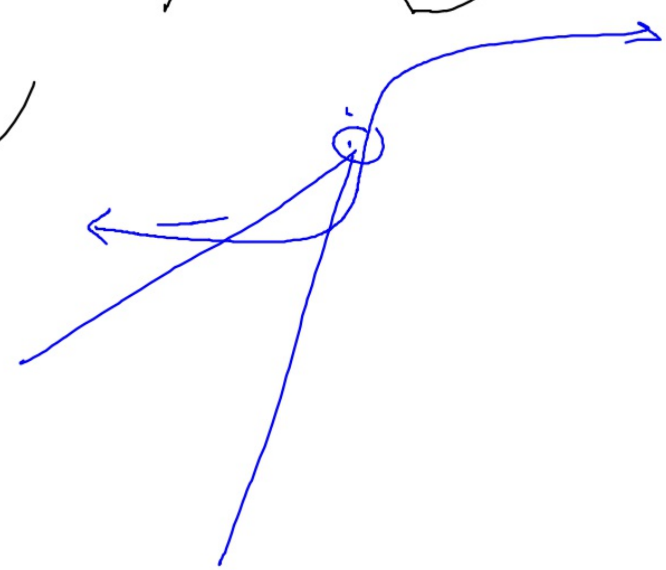
> A function can be described as "differentiable" on an interval if it is differentiable at every point in that interval.

Another way to think of it: $f(x)$ is differentiable at c if and only if $f\,'(x)$ is continuous at c.



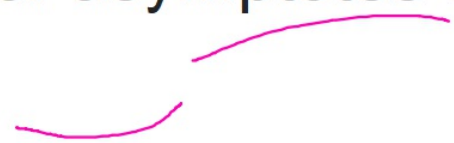




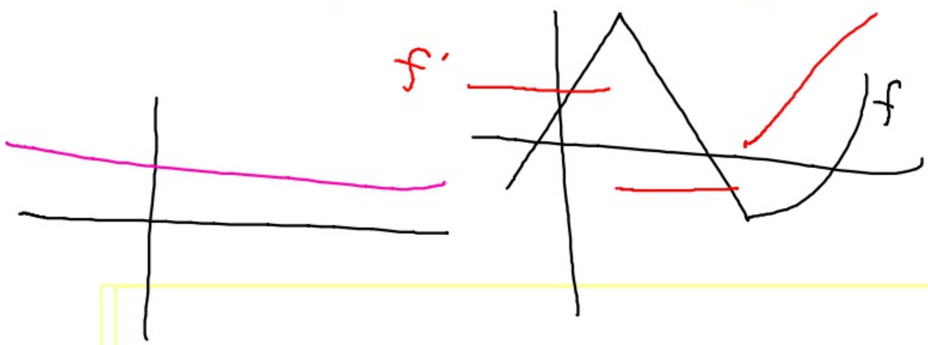


How to make a function differentiable

It must first be continuous!
Mr Pickle can't cross over holes or asymptotes or jumps.

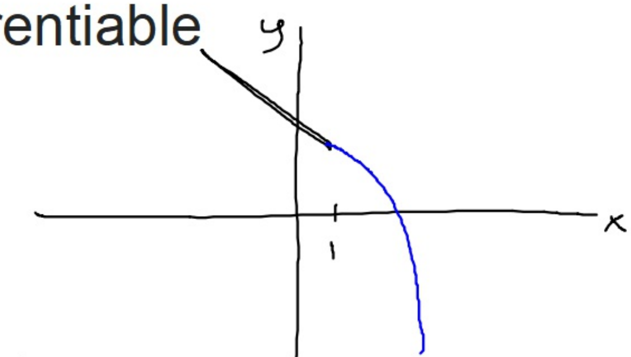


Then, the <u>derivative</u> must also be continuous
The slope must change smoothly



Find the values of a and b to make f differentiable

$$f(x) = \begin{cases} 5-2x, & x \le 1 \\ ax^2+bx, & x>1 \end{cases}$$



Continuity @ x=1

$$\lim_{x\to 1^-} f(x) = f(1) = \lim_{x\to 1^+} f(x)$$

$$5-2(1) =$$

$$3 = \boxed{3 = a+b}$$

$$f'(x) = \begin{cases} -2, & x \le 1 \\ 2ax+b, & x>1 \end{cases}$$

f' cont. @ x=1

$$\lim_{x\to 1^-} f'(x) = f'(1) = \lim_{x\to 1^+} f'(x)$$

$$-2 = -2 = 2(a)(1)+b$$

$$\boxed{-2 = 2a+b}$$

Solve system

$$\begin{cases} a+b=3 \\ 2a+b=-2 \end{cases}$$

$$-(2a+b=-2)$$

$$-1a + \cancel{0b} = 5$$

$$-a = 5$$

$$a = -5$$

$$-5+b=3$$

$$b=8$$