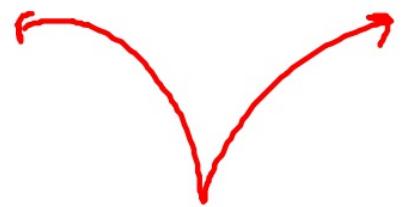


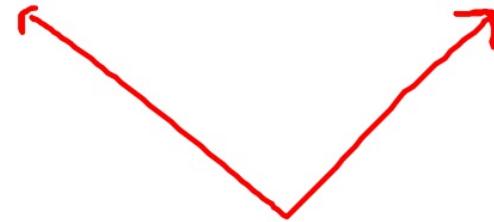
## AP Calculus

Continuing a discussion on differentiable functions

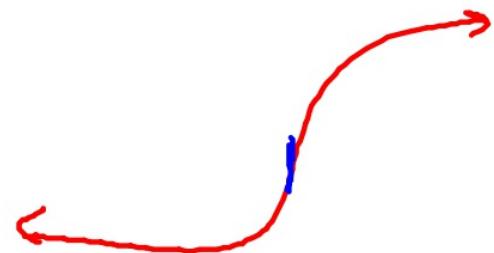
## Types of non-differentiable points



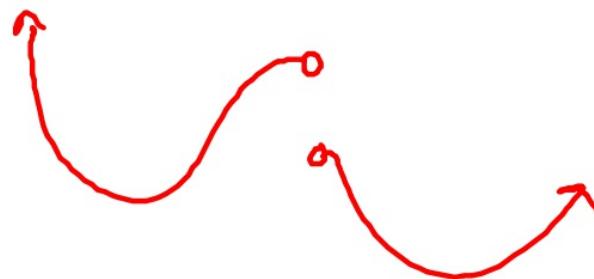
Cusp



Corner

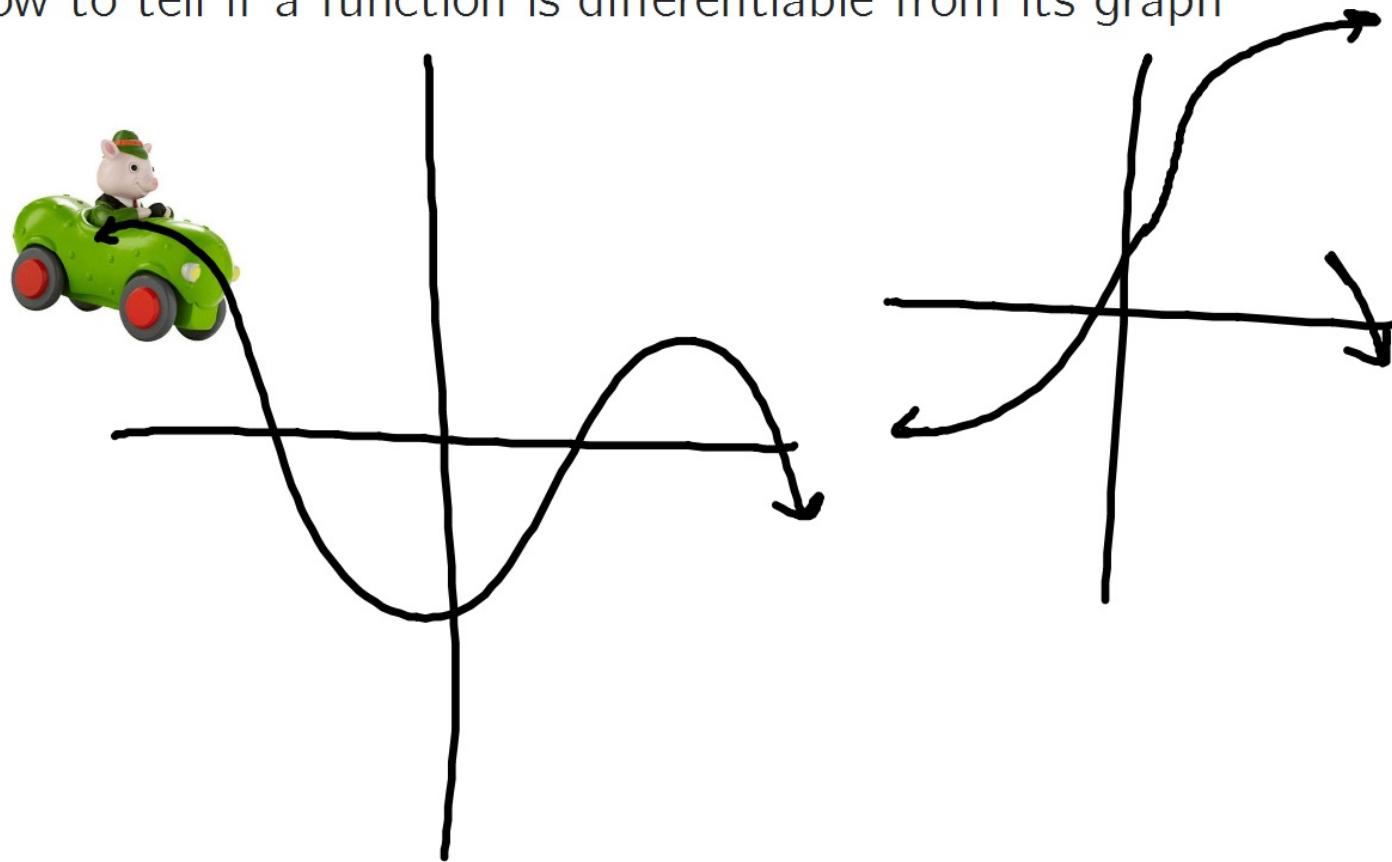


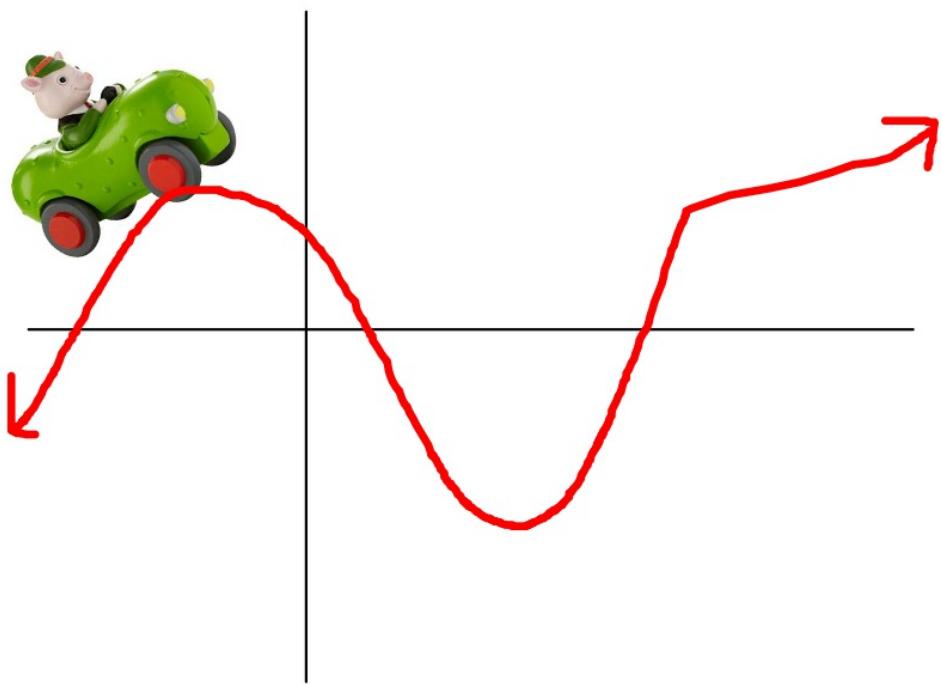
Vertical tangents



Discontinuities

How to tell if a function is differentiable from its graph





Differentiable function?

Good afternoon: Warm up in notebooks

$$\int 3x^2 - 4x + 4 \, dx \\ = x^3 - 2x^2 + 4x + C$$

1. If  $y' = 3x^2 - 4x + 4$ , what could be a possible equation for  $y$ ? How many possible answers are there?

$$y = x^3 - 2x^2 + 4x + 3$$

2. Find the limit:  $\lim_{\Delta x \rightarrow 0} \frac{-\cos(x+\Delta x) + \cos(x)}{\Delta x}$  " " " " " + 527.

$$\lim_{\Delta x \rightarrow 0} \frac{-\cos(x+\Delta x) - \cancel{-\cos(x)}}{\Delta x}$$

$x^3 + \cancel{2x^2}$

$= \sin(x)$

## Differentiable Function:

Verbally: A continuous function where every pt.  
has a unique tangent w/ defined slope

Graphically:

- must be continuous [no holes, no jumps,  
- no corners, no cusps, no no vertical asymptotes] tangents.]

Algebraically:

- $f$  is continuous

Vertical tangents.

- $f'$  is also continuous.

How to algebraically show a function is differentiable at a point:

- First show it is continuous at the point
- Then take the derivative and show that IT is continuous at the point

ex: Determine if  $f$  is differentiable everywhere.

$$f(x) = \begin{cases} x^2 - 4 & x < 0 \\ -\sin x + 2 & x \geq 0 \end{cases}$$

Disc.  $\lim_{x \rightarrow 0^-} f(x)$   $\lim_{x \rightarrow 0^+} f(x)$

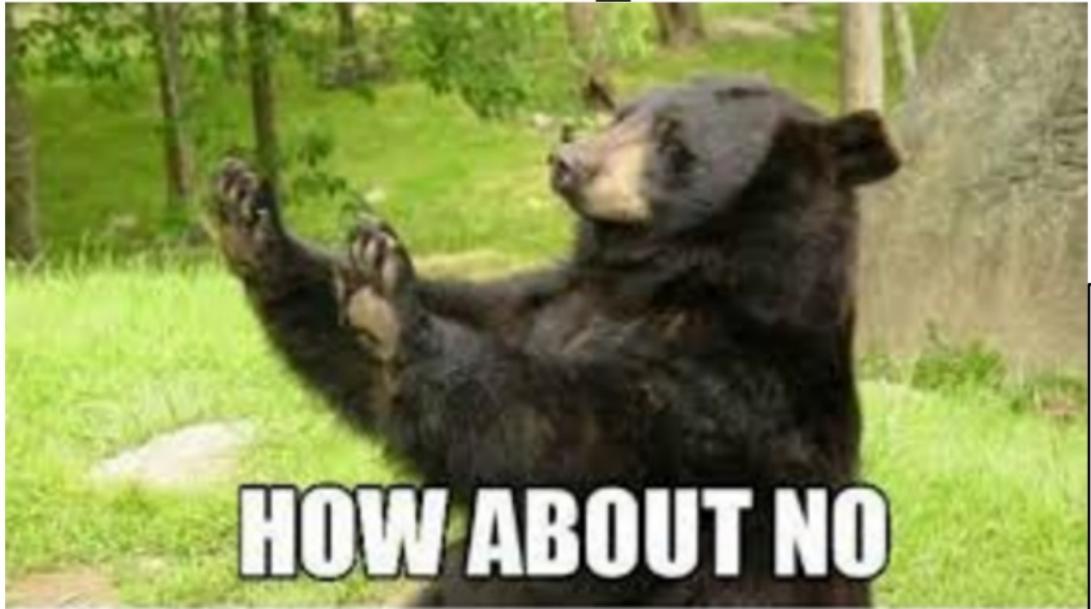
$$-4 \neq 2$$
$$f'(x) = \begin{cases} 2x & x < 0 \\ -\cos(x) & x \geq 0 \end{cases}$$

Product and Quotient Rules: things you gotta memorize -\_-

ex

Find  $dy/dx$  if  $y = x^2 \sin(x)$

$$dy/dx = 2x \cos(x)$$



The derivative of a product is NOT the product of the derivatives

(add to notes and then booklet)

## Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Alternate form:  $[fg]' = f'g + fg'$

ex:

Find  $f'(x)$  if  $f(x) = \underline{\underline{x^2}} \sin(x)$

$f: x^2$        $g: \sin(x)$

$f': 2x$        $g': \cos(x)$

$$f'(x) = f'g + fg'$$

$$2x \cdot \sin x + x^2 \cdot \cos x$$

$$2x \sin x + x^2 \cos x$$



Remember the recipe  
List out your ingredients!

$$y = (x^2 - 3x + 2)(x^2 - 4x + 1)$$

Find  $y'$

$$f: x^2 - 3x + 2 \quad g: x^2 - 4x + 1$$

$$f' = 2x - 3 \quad g' = 2x - 4$$

$$y' = f'g + fg'$$

$$(2x - 3)(x^2 - 4x + 1) + (x^2 - 3x + 2)(2x - 4)$$

$$\begin{aligned} & 2x^3 - 8x^2 + 2x \\ & - 3x^2 + 12x - 3 \end{aligned}$$

$$\begin{aligned} & 2x^3 - 6x^2 + 4x \\ & - 4x^2 + 12x - 8 \end{aligned}$$

$$2x^3 - 11x^2 + 14x - 3$$

$$2x^3 - 10x^2 + 16x - 8$$

$$4x^3 - 21x^2 + 30x - 11$$



Remember the recipe

List out your ingredients!

Quotient Rule...

Find  $f'(x)$  if  $f(x) = \frac{4x-4}{x^2-3}$

$$f'(x) = \frac{4}{2x}$$



Derivative of a quotient is NOT the quotient of the derivatives!

Quotient Rule      notes and booklet plz

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\left( \frac{f}{g} \right)' = \frac{f'g - fg'}{g^2}$$

Find  $f'(x)$  if  $f(x) = \frac{4x-4}{x^2-3}$

$$f: 4x-4 \quad g: x^2-3$$

$$f' = 4 \quad g' = 2x$$

$$\frac{dy}{dx} = \frac{4(x^2-3) - (4x-4)(2x)}{(x^2-3)^2}$$

$$\frac{4x^2-12 - [8x^2-8x]}{(x^2-3)^2}$$

$$\frac{4x^2-12-8x^2+8x}{(x^2-3)^2} \Rightarrow$$



Remember the recipe  
List out your ingredients!

$$\boxed{\frac{-4x^2 + 8x - 12}{(x^2-3)^2}}$$

$$f(x) = \frac{5-3x+x^2}{x+2}$$

$$f'(x)$$



Remember the recipe  
List out your ingredients!

Find the derivative of  $y=\tan(x)$

Homework over fall break -\_-

p. 105 #75-80 (solve graphically)  
#87-90 (solve algebraically)

p. 125: 3-33 (multiples of 3)

Reassessment