Good afternoon

No warm up today; check hw sols

87. polynomials are continuous, so IVT applies
f(1)=3.083 and f(2)=-2.667. 0 is between these, so by IVT, there is some c between 1 and 2 such that f(c)=0

88. polynomials are continuous, so IVT applies
f(0)= -3 and f(1)=3. 0 is between these, so by IVT, there is some c between 0 and 1 such that f(c)=0

89. polynomials and cosine are continuous, so IVT applies
f(0)=-3 and f(π)=8.87 . 0 is between these, so by IVT, there is some c between 0 and π such that f(c)=0

90. $f(x) = -\frac{5}{x} + \tan\left(\frac{\pi x}{10}\right)$ is continuous on the interval $[1, 4]$.

$f(1) = -5 + \tan\left(\frac{\pi}{10}\right) \approx -4.7$ and

$f(4) = -\frac{5}{4} + \tan\left(\frac{2\pi}{5}\right) \approx 1.8$. By the Intermediate Value Theorem, there exists a number c in $[1, 4]$ such that $f(c) = 0$.

95. c=3 (note that -4 is not in [0,5]
96. c=2 (note that 4 is not in [0,3]
97. c=2 (use long division to factor)
98. c=3 (note that 2 is not in [2.5,4]

112. Let $V = \frac{4}{3}\pi r^3$ be the volume of a sphere with radius r. V is continuous on $[5, 8]$. $V(5) = \frac{500\pi}{3} \approx 523.6$ and $V(8) = \frac{2048\pi}{3} \approx 2144.7$. Because $523.6 < 1500 < 2144.7$, the Intermediate Value Theorem guarantees that there is at least one value r between 5 and 8 such that $V(r) = 1500$. (In fact, $r \approx 7.1012$.)

98.) $f(x) = \frac{x^2+x}{x-1}$ $[2.5, 4]$ $f(c) = 6$

over Interval, $f(x)$ is Cont. IVT applies.

$f(2.5) = \frac{35}{6} = 5.\ldots$

$f(4) = \frac{20}{3} = 6.666\ldots$

$$\frac{x^2+x}{x-1} = \frac{6}{1}$$

$$\cancel{x-1}\left[\frac{x(x+1)}{\cancel{x-1}} = 6\right] x-1$$

$$x^2+x = 6x-6$$

$$x^2-5x+6 = 0$$

$$(x-3)(x-2) = 0$$

$x=3$ $x=2$

Wednesday's assessment:

similar to this homework + AP IVT questions handed out Friday [F-C4]

Remember that a function must be continuous for IVT to apply. Must state that before applying

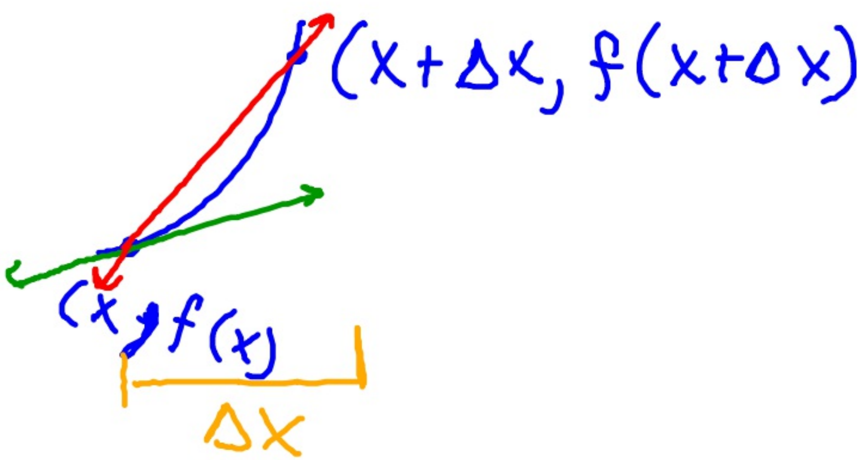
Review skills:

- Showing that a function is (dis)continuous [F-C1]
- Finding and justifying discontinuities [F-C3]

Limit Definition of Derivative

the slope of the tangent line at $(x, f(x))$ is given by

$$\underbrace{f'(x)}_{\text{"f prime"}} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



What is a "derivative"? (Notes

Leibniz

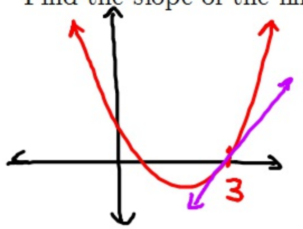
- Slope of the line tangent to a curve
- Instantaneous rate of change (vs. average rate of change)
- "Velocity" (as opposed to position)
- limit of the difference quotient
- slope at 1 point
- 'curviness' of a function at one point

Notation: be comfortable with all of these!

$f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{d}{dx}[\;]$ $\quad D_x[\;]$

MY FIRST DERIVATIVE

Find the slope of the line tangent to $f(x)=x^2-4x+3$ at the point (3,0)



$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$-(x^2-4x+3)$$

$$\frac{(x+\Delta x)^2 - 4(x+\Delta x)+3 - x^2+4x-3}{\Delta x}$$

$$\frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2-\cancel{4x}-4\Delta x\cancel{+3}-\cancel{x^2}+\cancel{4x}-\cancel{3}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x\Delta x+(\Delta x)^2-4\Delta x}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x+\Delta x-4)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 2x+\cancel{\Delta x}-4$$

$$f'(x) = 2x-4$$

$$f'(3) = 2(3)-4 = 2$$

HW: