Good afternoon: assessments are being passed back

Strengths:
- Linear Approximation
- Horizontal/Vertical Tangents
- Differentiability

Weaknesses
- Chain Rule
- Tangent Line Eq
- Motion

Peer experts:
- #1 and 2: Mak
- #3: Ben
- #11-12: Virginia
- #15-16: Sarah

Visibly Random Grouping

#1-2 Mak

1. The position, in meters, of a bandit moving along a straight path is given by the differentiable function $s(t) = -2t^3 + 14t^2 - 32t + 24$. Find all times t where the bandit is at rest.

pos
vel
acc
jerk

$-6t^2 + 28t - 32$

$-(6t^2 - 28t + 32)$

$-(3t - 8)(2t - 4)$

32 1
16 2
8 4

$3t - 8 = 0$, $3t = 8$, $t = \frac{8}{3}$

$2t - 4 = 0$, $2t = 4$, $t = 2$

$v(t) = 0$ ✓

$v(0)$ ✗

t @ rest @ $\frac{8}{3}$, 2

2. The position, in feet, of a crazy train being robbed by some rough pardners moving along a straight track is given by the differentiable function $x(t) = -\cos 2t + \sin 6t$ where t is measured in seconds. Find the acceleration of the train at t=0. Include units in your answer.

$a(0)$

$-\cos 2t + \sin 6t$

$\sin 2t \cdot 2 + \cos 6t \cdot 6$

$2\sin 2t + 6\cos 6t$

$d(t)$ $2\cos 2t \cdot 2 + -6\sin 6t \cdot 6$

$4\cos 2t - 36\sin 6t$

$4\cos 2(0) - 36\sin(0)t$

$4 - 0$

4 ft/s^2

#3 Ben

3. Graphed below is the velocity, in meters per second, as a function of time over the first 12 seconds of a horsey going just absolutely bananas.

- When does the horsey change direction?

when the line crosses the x axis at 4 and 11

- When is the horsey at its greatest speed?

at x = 2

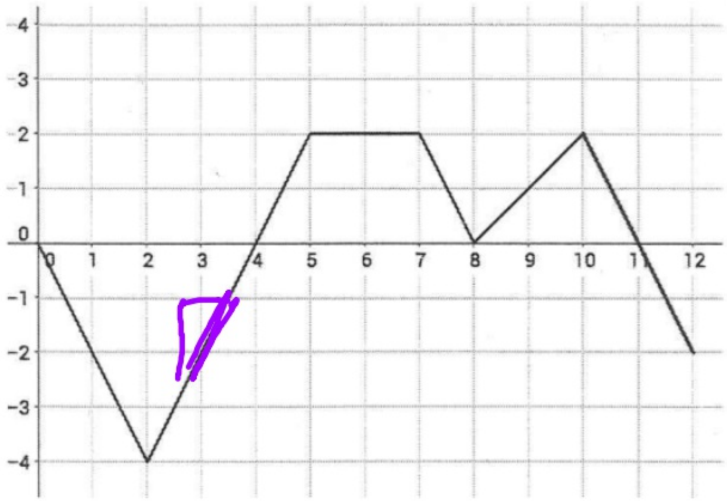
- At time t=1, is the horsey slowing down or speeding up? Explain.

horsey is speeding up because going a negative velocity means that you are going backwards if it is below zero

- Calculate the acceleration at t=3 sec. Include units.

$v/t = 2$ meters/sec^2

ney. accel.



Above graph shows velocity $v(t)$

#11-12: Virginia

SELF: D-CD7

11. Write the equation of the line tangent to $y = (1 - \frac{1}{2}x)^3$ where x=4

$y = -1$

$m = -3/2$

$y = (1 - \frac{1}{2}(4))^3$

$y = (1-2)^3$

$y = -1$

$3(1 - \frac{1}{2}x)^2 \cdot -\frac{1}{2}$

$-3/2 (1 - 1/2 x)^2$

$-3/2 (-1)^2 = -3/2$

$y + 1 = -3/2 (x - 4)$

$y + 1 = \frac{-3}{2}x + 6$

$-1 \qquad -1$

$\boxed{y = -3/2 x + 5}$

12. Write the equation of a line with a slope of 4 that is tangent to $y = -2x^2 - 4x + 1$.

$\boxed{m = 4 \quad x = -2 \quad y = 1}$

$4 = -4x - 4$

$+4 \qquad +4$

$8 = -4x$

$\overline{-4}$

$x = -2$

$-2(-2)^2 - 4(-2) + 1$

$-8 + 8 + 1$

$y = 1$

$y - 1 = 4(x + 2)$

$y - 1 = 4x + 8$

$+1 \qquad +1$

$\boxed{y = 4x + 9}$

#15-16: Sarah

15. Find $Q'(4)$ if $Q(t) = \sqrt{2t^2 - 7}$

$(2t^2-7)^{1/2}$

$Q'(t) = \frac{1}{2}(2t^2-7)^{-1/2} \cdot \frac{4t}{1}$

$\frac{4t}{2\sqrt{2t^2-7}}$

$\frac{16}{2\sqrt{32-7}}$

$\frac{16}{2\sqrt{25}} = \frac{16}{10} = \boxed{\frac{8}{5}}$

16. Suppose $r = \sec^2 4\theta$. Find $\frac{dr}{d\theta}$

$[\sec 4\theta]^2$

$2\sec(4\theta) \cdot \sec(4\theta)\tan(4\theta) \cdot 4$

$\boxed{8\sec(4\theta) \cdot \sec(4\theta)\tan(4\theta)}$

sin	cos
cos	-sin
tan	$\sec^2$
csc	-csc cot
cot	$-\csc^2$
sec	sec tan

D-C2: Interpreting the Derivative

SELF: D-C2

4. A tree is planted into black, loamy soil. Its height, in centimeters, as a function of time can be modeled by the differentiable function $H(t)$ for time t in days. Suppose that $\left.\frac{dH}{dt}\right|_{t=6} = 0.215$. Explain the meaning of this expression using correct units.

$$\frac{dH}{dt} = \frac{cm}{day}$$

.215 cm/day

5. It's Thanksgiving and a big juicy turkey/tofurkey is pulled out of the oven and is cooling off. Its temperature F in degrees Fahrenheit is a function of the time t in minutes it has been pulled out of the oven.

   a. Explain why $F'(t)$ must be negative.

F' $\quad \frac{dF}{dt} = \frac{°F}{min}$ -3.4

   b. Interpret, using correct units, the meaning of $F'(5) = -3.4$

Rates ≠ Values

D-AD0: L'Hopital's Rule

SELF: D-AD0

Evaluate each limit using l'Hôpital's Rule.

8. $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = \frac{0 + \sin x}{1} = \frac{0}{1} = \underline{0}$

9. $\lim\limits_{x\to 1} \frac{x^7-1}{x-1} \longrightarrow \frac{7x^6}{1} \quad \frac{7}{1} \longrightarrow 7$

10. $\lim\limits_{x\to \infty} \frac{4x^2}{e^x} \Rightarrow \frac{8x}{e^x} \Rightarrow \frac{8}{e^x} = \frac{8}{\infty} = 0$

Today's topic: Implicit Differentiation

We can take derivatives of all function types...right?

- Constant
- Linear
- Quadratic
- Cubic and other Polynomials
- Rational functions
- Exponential
- Logarithmic
- Trig
- Inverse Trig

Composite Functions

$\sin(3x)$

Explict vs Implicit
Functions

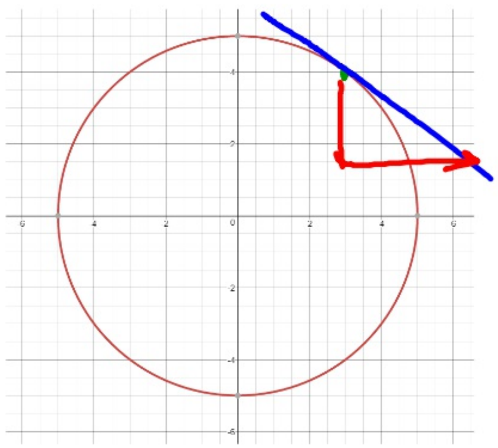
$$\overset{dep}{y} = 3\overset{ind}{x} + 5 \quad \text{vs.} \quad y - 3x = 5$$

∴ explicit function (solved for y)

implicit function (not solved for y)

MY FIRST IMPLICIT DERIVATIVE

Find the slope of the line tangent to $x^2+y^2=25$ at the point (3,4).



$$x^2+y^2=25$$

$$\frac{d}{dx}\left(x^2+y^2\right)=\frac{d}{dx}(25)$$

$$2x+2y\cdot\frac{dy}{dx}=0$$

Chain Rule

$$\frac{2y\frac{dy}{dx}}{2y}=\frac{-2x}{2y}$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\left.\frac{dy}{dx}\right|_{(3,4)}=-\frac{3}{4}$$

① use the deriv. as an operator.

→ take deriv of both sides

② Chain on $\frac{dy}{dx}$ to every y-derivative

③ Isolate dy/dx

think of "y" as $y(x)$ (function notation)

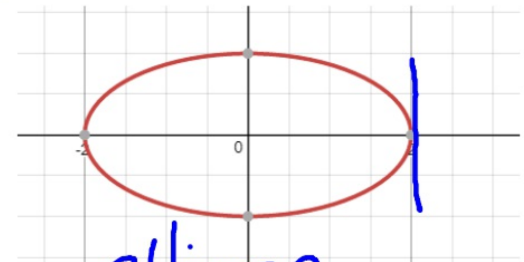
thus, $\frac{d}{dx}\, y(x) = y'(x) \;\; \frac{dy}{dx}$

Nutshell:

tack on $\frac{dy}{dx}$ when taking derivative of the y terms.

Show that $x^2+4y^2=4$ has a vertical tangent when $x=2$

$$\frac{d}{dx}\left(x^2+4y^2\right)=\frac{d}{dx}\left(4\right)$$



ellipse

$$2x+8y\frac{dy}{dx}=0$$

$$8y\frac{dy}{dx}=-2x \longrightarrow \frac{dy}{dx}=\frac{-2x}{8y}=\boxed{\frac{-x}{4y}}$$

Derivative:

$$\frac{dy}{dx}=\frac{-x}{4y}$$

$x^2+4y^2=4$

$x=2$ ... what's y?

$4^2+4y^2=4$

$4y^2=0 \rightarrow y=0$

$\Rightarrow (2,0) \rightarrow$ plug into $\frac{dy}{dx}$

$$\frac{-2}{4(0)}=\frac{-2}{0}\rightarrow \text{undefined!}$$

$\therefore$ vertical tangent.

$3\sin(y)=x^2$

make explicit

$\frac{d}{dx}3\sin(y) = \frac{d}{dx}x^2$

$3\cos(y)\cdot\frac{dy}{dx} = 2x$

$$\boxed{\frac{dy}{dx} = \frac{2x}{3\cos y}}$$

easy!

$\sin y = \frac{x^2}{3}$

$y = \sin^{-1}\left(\frac{x^2}{3}\right)$

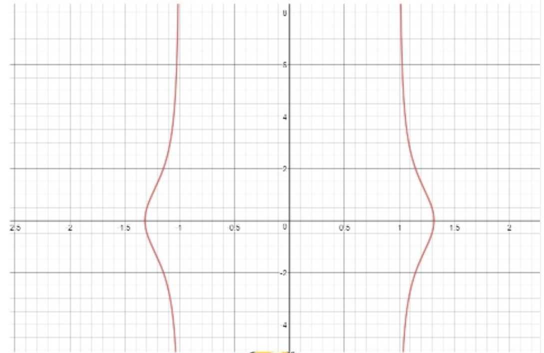
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\frac{x^4}{9}}}\cdot\frac{2x}{3}$$

hard!

product rule

$$x^4 + \underbrace{x^2}_{f}\underbrace{y^2}_{g} - y^2 = 3$$

$$4x^3 + 2x\cdot 2y\frac{dy}{dx} \qquad - 2y\frac{dy}{dx} = 0$$



$$4x^3 + \underbrace{2x}_{f'}\underbrace{y^2}_{g} + \underbrace{x^2}_{f}\cdot \underbrace{2y\frac{dy}{dx}}_{g'} - 2y\frac{dy}{dx} = 0$$

$$2x^2y\frac{dy}{dx} - 2y\frac{dy}{dx} = -4x^3 - 2xy^2$$

$$\frac{\frac{dy}{dx}\left(2x^2y - 2y\right)}{2x^2y - 2y} = \frac{\text{'' \quad ''}}{2x^2y - 2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-4x^3 - 2xy^2}{2x^2y - 2y}}$$

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

a.) $2y \cdot \frac{dy}{dx} = 0 + 1 \cdot y + x \cdot \frac{dy}{dx}$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx}(2y - x) = y \rightsquigarrow \boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$

b.) Set $\frac{dy}{dx} = \frac{1}{2}$

$\frac{y}{2y-x} = \frac{1}{2} \Rightarrow 2y = 2y - x \rightarrow x = 0$ plug into original funct.

$y^2 = 2 + xy$

$(0, \sqrt{2})$
$(0, -\sqrt{2})$ $\leftarrow$ $y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

c.) Horiz. Tangent line $\Rightarrow$ Numerator of $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y}{2y-x} \rightarrow y = 0 \xrightarrow{\text{plug into}}$ original function

$y^2 = 2 + xy$
$0 = 2 + 0 \cdot x \rightarrow 0 = 2$?! Nonsense!

Thus, y is never 0,
so $\frac{dy}{dx} \neq 0$, so no H.T.

HW
'mixed review'
due 11/29