

Good afternoon:

We will have a mini-lesson on using the IVT shortly, have notes out

Assessments are being passed back, look over them with partners, ask questions and learn :)



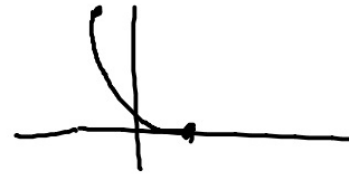
Using the IVT

Find the value(s) of  $c$  in  $[-2, 3]$  guaranteed to exist by the IVT such that  $f(c) = 2$  for  $f(x) = x^2 - 5x + 6$

$f(x)$  is cont. b/c polynomial  $\Rightarrow$  IVT applies.

$$f(-2) = (-2)^2 - 5(-2) + 6 = 20$$

$$f(3) = 3^2 - 5(3) + 6 = 0$$



Since  $20 \geq 2 \geq 0$ , some  $c \in [-2, 3]$  exists.

$$\underset{-2}{f(c)} = 2 = c^2 - 5c + \underset{-2}{6}$$

$$0 = c^2 - 5c + 4$$

$$0 = (c - 4)(c - 1)$$

$\downarrow$   $\downarrow$   
 ~~$c = 4$~~   $c = 1$   
not in  $[-2, 3]$

Find the value(s) of  $c$  in  $[0,7]$  guaranteed to exist by the IVT such that  $f(c)=28$  for  $f(x)=x^2+6x+1$

$$f(0) = 1$$
$$f(7) = 92$$

$$92 > 28 > 1$$

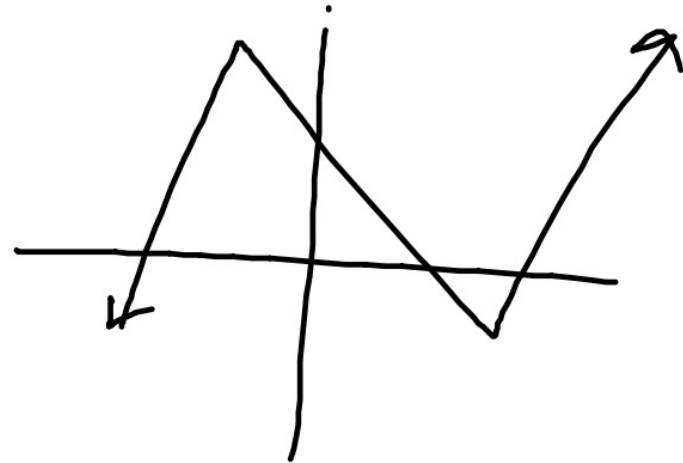
$$f(c) = c^2 + 6c + 1 = 28$$

$$c^2 + 6c - 27 = 0$$

$$(c+9)(c-3) = 0$$

$$\cancel{c = -9} \quad \textcircled{c = 3}$$

## IVT AP Practice



Good afternoon: warm up, do #1,3-5 on the handout from DS

**You will need a device today  
(can use computers in back if needed)**

1.

Let  $f$  be a continuous function on the closed interval  $[-3, 6]$ . If  $f(-3) = -1$  and  $f(6) = 3$ , then the Intermediate Value Theorem guarantees that

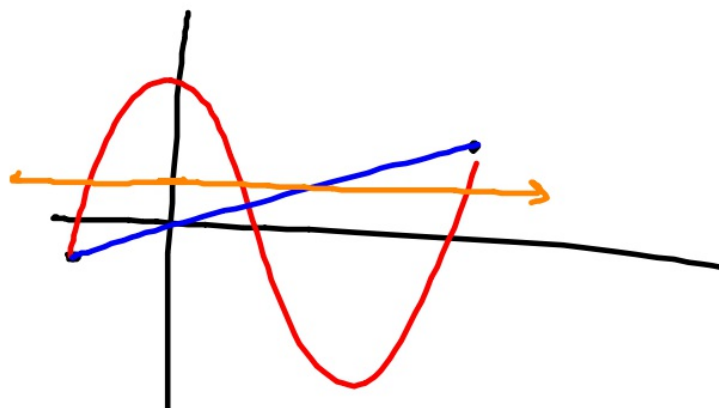
(A)  $f(0) = 0$

(B)  $f'(c) = \frac{4}{9}$  for at least one  $c$  between  $-3$  and  $6$

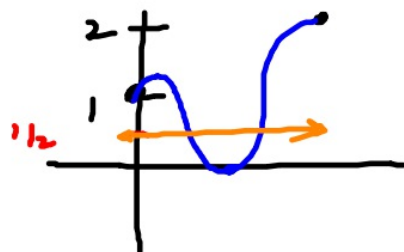
~~(C)  $-1 \leq f(x) \leq 3$  for all  $x$  between  $-3$  and  $6$~~

**(D)  $f(c) = 1$  for at least one  $c$  between  $-3$  and  $6$**

(E)  $f(c) = 0$  for at least one  $c$  between  $-1$  and  $3$



3.



|        |   |     |   |
|--------|---|-----|---|
| $x$    | 0 | 1   | 2 |
| $f(x)$ | 1 | $k$ | 2 |

The function  $f$  is continuous on the closed interval  $[0, 2]$  and has values that are given in the table above. The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval  $[0, 2]$  if  $k =$

(A) 0

(B)  $\frac{1}{2}$

(C) 1

(D) 2

(E) 3

4.

Let  $g$  be a continuous function on the closed interval  $[0,1]$ . Let  $g(0) = 1$  and  $g(1) = 0$ . Which of the following is NOT necessarily true?

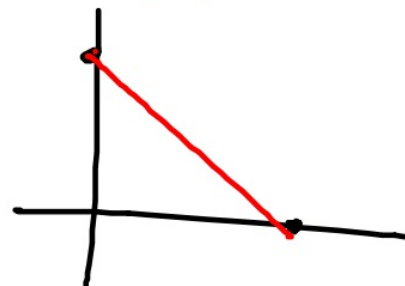
(A) There exists a number  $h$  in  $[0,1]$  such that  $g(h) \geq g(x)$  for all  $x$  in  $[0,1]$ .

(B) For all  $a$  and  $b$  in  $[0,1]$ , if  $a = b$ , then  $g(a) = g(b)$ .

(C) There exists a number  $h$  in  $[0,1]$  such that  $g(h) = \frac{1}{2}$ .

(D) There exists a number  $h$  in  $[0,1]$  such that  $g(h) = \frac{3}{2}$ .

(E) For all  $h$  in the open interval  $(0,1)$ ,  $\lim_{x \rightarrow h} g(x) = g(h)$ .

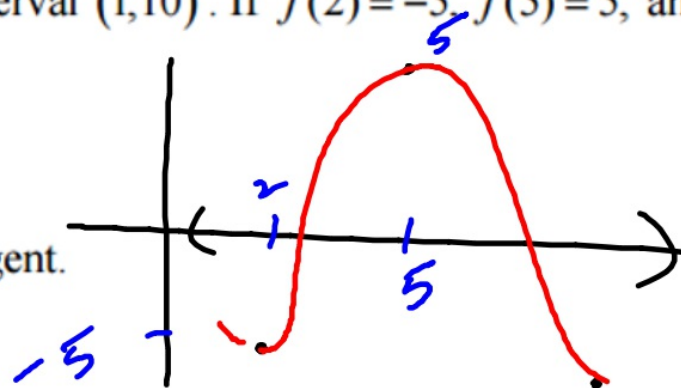




5.

Let  $f$  be a function that is differentiable on the open interval  $(1,10)$ . If  $f(2) = -5$ ,  $f(5) = 5$ , and  $f(9) = -5$ , which of the following must be true?

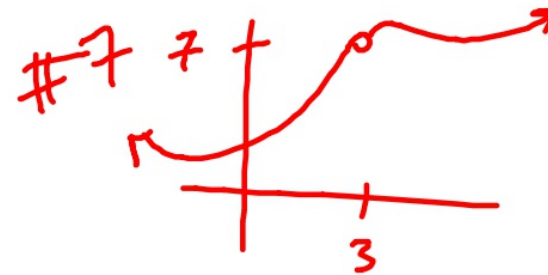
- ☒ I.  $f$  has at least 2 zeros.
- ☒ II. The graph of  $f$  has at least one horizontal tangent.
- III. For some  $c$ ,  $2 < c < 5$ ,  $f(c) = 3$ .



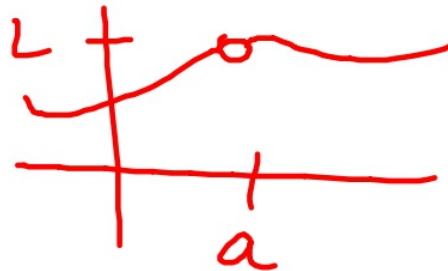
- (A) None
- (B) I only
- (C) I and II only
- ☒ (D) I and III only
- ☒ (E) I, II, and III

Pass eggs to person nearest to the door  
Get out limits packet, submit answers at

[bit.ly/limitspack](http://bit.ly/limitspack)

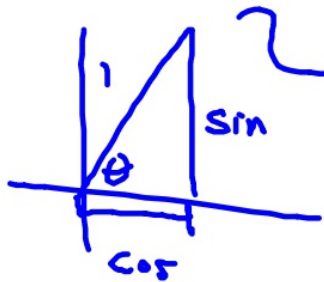


# 3.)  $\lim_{x \rightarrow a} f(x) = L$   
(E)



$$\textcircled{11} \lim_{x \rightarrow 0} \frac{1 - \cos x}{2 \sin^2 x} = \frac{1 - \cos x}{2(1 - \cos^2 x)} = \frac{1 - \cos x}{2(1 + \cos x)(1 - \cos x)}$$

$$\Rightarrow \sin^2 x + \cos^2 x = 1$$



$$\sin^2 = 1 - \cos^2$$

$$\lim_{x \rightarrow 0} \frac{1}{2(1 + \cos x)} = \frac{1}{2(1+1)} = \frac{1}{4}$$

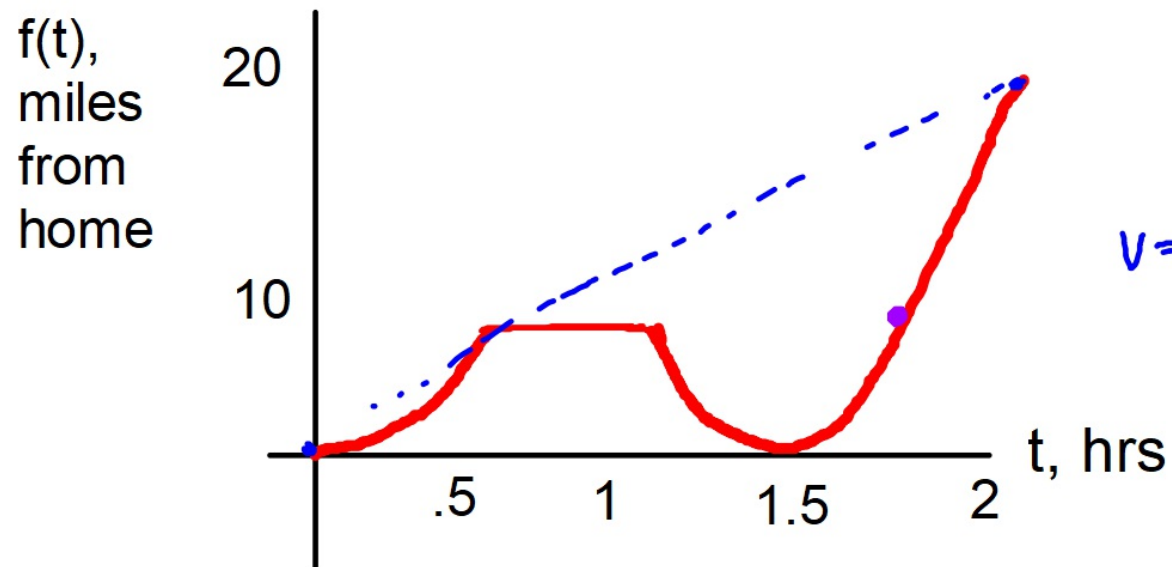
18

$$f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & , x \neq 2 \\ K & , x = 2 \end{cases}$$

$$\lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2x+5} + \sqrt{x+7}}{\sqrt{2x+5} + \sqrt{x+7}}$$

$$\frac{2x+5 - (x+7)}{(x-2)(\sqrt{2x+5} + \sqrt{x+7})} \Rightarrow \frac{2x+5 - \cancel{x} - 7}{(\cancel{x-2})(\sqrt{2x+5} + \sqrt{x+7})}$$

## Beginning the Derivative



Average velocity?

over these  
2 hrs.

$$v = \frac{\Delta S}{\Delta t} = \frac{20 \text{ miles}}{2 \text{ hrs}} = \underline{10 \text{ mph.}}$$

## Slope Formula Through the Ages

Algebra I

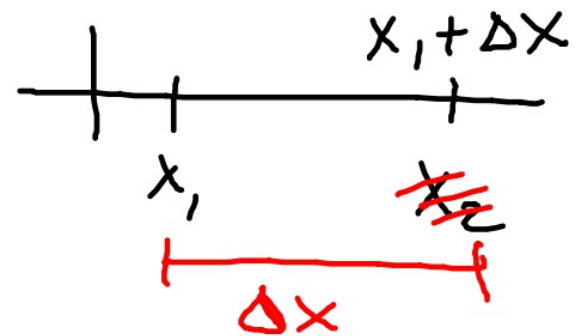
$$\frac{y_2 - y_1}{x_2 - x_1}$$

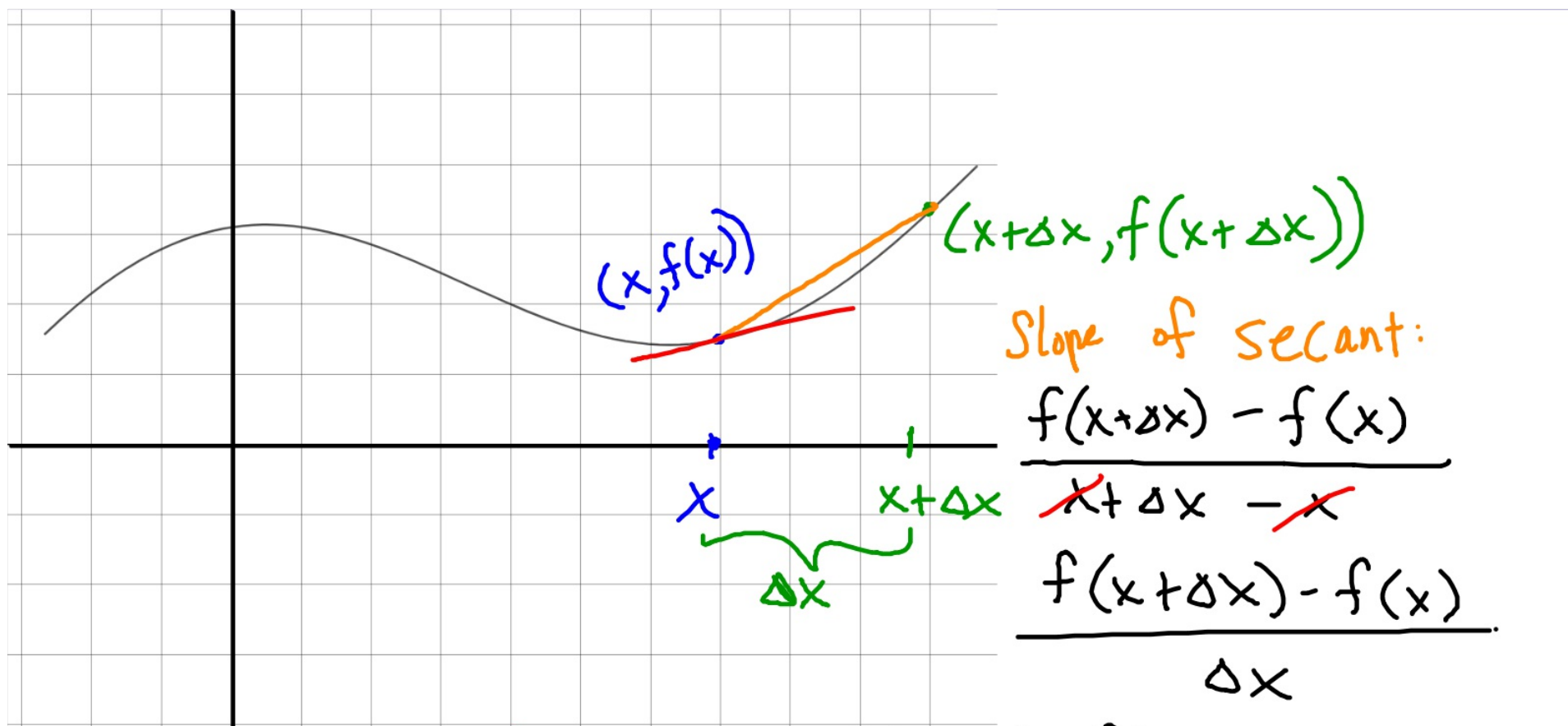
Function Notation

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

"  
Difference  
Quotient"

$$\left\{ \frac{f(x + \Delta x) - f(x)}{\Delta x} \right.$$





Derivative = Slope of tangent line  $\Rightarrow \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Limit definition of derivative

ADD TO BOOKLETS

The slope of a line tangent to a function  $f$  at  $(x, f(x))$  is given by

$$\underbrace{f'(x)}_{\text{"f prime of x"}} = \underbrace{\frac{df}{dx}}_{\text{"f prime of x"}} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



## What *is* a derivative?

- Slope of the line tangent to a curve
- Instantaneous rate of change (vs. average rate of change)
- "Velocity" (as opposed to position)
- limit of the difference quotient
- slope at 1 point
- 'curviness' of a function at one point

and much much more!!!