What is an inverse trig function? (notes)

FYI: Hw problems: #556-572 (evens), 576, 580, 582, 586, 589, 604-606, 609, 610 (D-AD2)

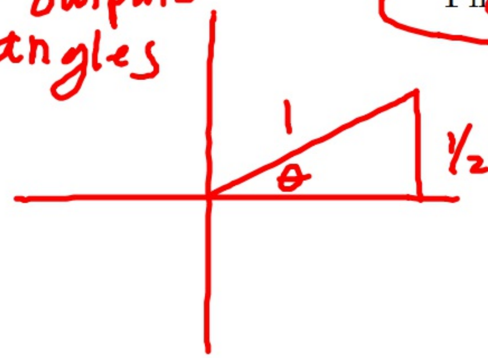
What is arcsin(1/2) asking?

it outputs angles

$\sin^{-1}\left(\frac{1}{2}\right)$

$\arcsin\left(\frac{1}{2}\right)$

"What angle has a sine of $\frac{1}{2}$?"



Find arctan(tan(π/2)) = π/2

sin(angle) = ratio

arcsin(ratio) = angle

Inverse Trig Derivatives: (Notes)

Question: if $y = \sin^{-1}(x)$, what is dy/dx?

(angle → y; ratio → x)

$$\frac{d}{dx} x = \frac{d}{dx} \sin(y)$$

$$\frac{1}{\cos(y)} = \frac{\cos(y)\frac{dy}{dx}}{\cos(y)}$$

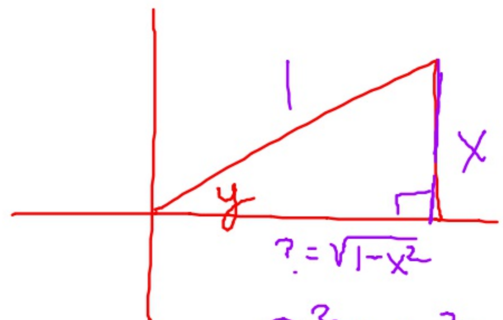
$$\frac{1}{\cos(y)} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$

$$\frac{d}{dx}\cos(x^2)$$

$$-\sin(x^2)\frac{d}{dx}(x)^2$$

$$-\sin(x^2)\,2x$$



$$\sin(y) = x$$

$$?^2 + x^2 = 1$$

$$?^2 = 1 - x^2$$

$$? = \sqrt{1-x^2}$$

$$\cos(y) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

Add to booklet: (this is the last of the derivative rules! :D)

Inverse Trig Derivatives

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} \qquad \frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\operatorname{arccsc} x = -\frac{1}{x\sqrt{x^2-1}}$$

Examples (notes)

$y = \arctan(\cos(x))$. Find dy/dx.

$$\frac{dy}{dx} = \frac{1}{1+\cos^2(x)} \cdot -\sin(x) = \frac{-\sin(x)}{1+\cos^2(x)}$$

$f(x) = \arcsin(2x)$. Find $f'(x)$.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

$y = \csc^{-1}(\pi x)$

$$\frac{dy}{dx} = \frac{-1}{\pi x\sqrt{(\pi x)^2-1}} \cdot \pi$$

$$\frac{-\pi}{\pi x\sqrt{\pi^2 x^2-1}}$$

$$\frac{-1}{x\sqrt{\pi^2x^2-1}}$$

Mixed review

$\frac{d}{dx}\ln(x) = \frac{1}{x}$

Take the derivative: $y = x\ln(x^2)$

$f = x$, $g = \ln(x^2)$

$f' = 1$

$g' = \frac{1}{x^2}\cdot\frac{2x}{1} \Rightarrow \frac{2x}{x^2} = \frac{2}{x}$

$f'g + fg'$

$1\cdot\ln(x^2) + x\cdot\frac{2}{x}$

$\ln x^2 + 2$

$\log_b x = a \Leftrightarrow b^a = x$

ex

$\ln e^{x^2}$

$\log_e e^{x^2}$

x^2

$\log 100 = 2$

$y = (\sin x)^x$ Find y'.

If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$

(D) e^{-2x} (E) $2e^{-2x}$

$$\frac{d}{dx}\cos^2(x^3) =$$

(A) $6x^2\sin(x^3)\cos(x^3)$

(B) $6x^2\cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2\sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

If $f(x)=\dfrac{e^{2x}}{2x}$, then $f'(x)=$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$

If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

(E) $-4(\cos x)(\sin x)$