2.10 Tangents, Normals, and Continuity (Revisited)

491. Find the equation of the tangent line to the curve $y = \sqrt{x^2 - 3}$ at the point $(2, 1)$.

492. Find the equation of the normal line to the curve $y = (3x - 1)^2(x - 1)^3$ at $x = 0$.

493. Find the equation of the tangent line to the curve $y = \sqrt{3x - 1}$ that is perpendicular to the line $3y + 2x = 3$.

494. Find the equation of the normal line to the curve $y = x\sqrt{25 + x^2}$ at $x = 0$.

495. Find the equation of the tangent line to the curve $y = \dfrac{2 - x}{5 + x}$ at $x = 1$.

496. Find the equation of the normal line to the curve $y = \dfrac{5}{(5 - 2x)^2}$ at $x = 0$.

497. Find the equation of the tangent line to the curve $y = 3x^4 - 2x + 1$ that is parallel to the line $y - 10x - 3 = 0$.

498. The point $P(3, -2)$ is not on the graph of $y = x^2 - 7$. Find the equation of each line tangent to $y = x^2 - 7$ that passes through P.

For the following six problems, determine if f is differentiable at $x = a$.

① f is cont.
② f' is cont.

499. $f(x) = |x + 5|$; $a = -5$

500. $f(x) = \begin{cases} x + 3 & x \le -2 \\ -x - 1 & x > -2 \end{cases}$ $a = -2$

501. $f(x) = \begin{cases} 2 & x < 0 \\ x - 4 & x \ge 0 \end{cases}$ $a = 0$

502. $f(x) = \begin{cases} -2x^2 & x < 0 \\ 2x^2 & x \ge 0 \end{cases}$ $a = 0$

503. $f(x) = \begin{cases} x^2 - 5 & x < 3 \\ 3x - 5 & x \ge 3 \end{cases}$ $a = 3$ $f' = \begin{cases} 2x, & x < 3 \\ 3, & x \ge 3 \end{cases}$

504. $f(x) = \begin{cases} \sqrt{2 - x} & x < 2 \\ (2 - x)^2 & x \ge 2 \end{cases}$ $a = 2$

505. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and at $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivatives of the following combinations of f and g at the given value of x.

a) $3f(x) - g(x)$, $x = -1$

b) $[f(x)]^3[g(x)]^3$, $x = 0$

c) $g(f(x))$, $x = -1$

d) $f(g(x))$, $x = -1$

e) $\dfrac{f(x)}{g(x) + 2}$, $x = 0$

f) $g(x + f(x))$, $x = 0$

Number rules the universe. —*Pythagoras*

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$y = \sqrt{x^2-3}$ @ $(2,1)$

$y = (x^2-3)^{1/2}$

$\frac{dy}{dx} = \cancel{\frac{1}{2}}(x^2-3)^{-1/2} \cdot \cancel{2}x$

$x^1(x^2-3)^{-1/2}$

$\frac{dy}{dx} = \frac{x}{\sqrt{x^2-3}} = \frac{2}{\cancel{\sqrt{4-3}}\ 1} = 2$

$y - 1 = 2(x-2)$

492. normal $y=\underbrace{(3x-1)^2}_{f}\underbrace{(x-1)^3}_{g}$ $x=0$.

$\frac{d}{dx}f(g(x))$

$= f'(g(x))g'(x)$

$f' = 2(3x-1)\cdot 3$

$6(3x-1)$

$f' = 18x-6$

$g' = 3(x-1)^2\cdot 1$

$g' = 3(x-1)^2$

$f'g + fg'$

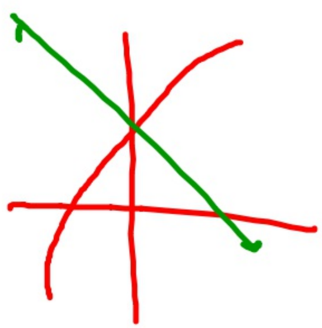
$$\frac{dy}{dx} = (18x-6)(x-1)^3 + (3x-1)^2\cdot 3(x-1)^2$$

$$(-6)(-1)^3 + \cancel{(-1)^2}\cdot 3\cancel{(-1)^2}$$

$$6 + 3 = 9$$ ← slope of tan line

so $\perp$ line

$\Rightarrow -\frac{1}{9}$ $(0,-1)$



$f(0) = -1$

$$\boxed{y+1 = -\frac{1}{9}(x)}$$