D-AD17 Big Ol' Practice Assessment

2^{nd} Deriv. of position

1. The position, in feet, of a particle moving along a straight path is given by the differentiable function $x(t) = \sin 2t - \cos 4t$ where t is measured in seconds. Find the acceleration of the particle at $t=0$. Include units in your answer.

$x'(t) = v(t) = \cos(2t)\cdot 2 - -\sin(4t)\cdot 4 = 2\cos(2t) + 4\sin(4t)$ ← velocity

velocity

$a(t) = x''(t) = v'(t) = 2\cdot -\sin(2t)\cdot 2 + 4\cos(4t)\cdot 4 = -4\sin(2t) + 16\cos(4t)$ ← accel

$a(0) = -4\sin(2\cdot 0) + 16\cos(4\cdot 0) = -4\sin(0) + 16\cos(0)$
$-4\cdot 0 + 16\cdot 1$ → $\boxed{16\ \text{ft/s/s}}$

2. The position, in feet, of a particle moving along a straight path is given by the differentiable function $s(t) = -t^3 + 5t^2 - 7t + 3$. Find all times t where the particle is at rest. → means velocity = 0

velocity? deriv. of position.

$v(t) = s'(t) = -3t^2 + 10t - 7$

at rest:

$v(t) = 0 = -3t^2 + 10t - 7$

$3t^2 - 10t + 7 = 0$

$(3t - 1)(t - 7) = 0$

$3t - 1 = 0$ → $t = 1/3$

$t - 7 = 0$ → $t = 7$

Particle @ rest when $t = 1/3$ and $t = 7$

3. A person is hiking in a national park and their velocity in meters per minute is graphed below.

When does the person change direction?

sign change in velocity

@ t = 30 min

When is the person slowing down?

Velocity and accel. (slope of vel.) have diff sign.

(15, 30) and (40, 60)

When is the person walking at their greatest speed?

largest absolute velocity

From t = 10 to t = 15

Calculate the acceleration at t=20min.

slope of vel.

$a(t) = \frac{dv}{dt} = \frac{-20\ \text{m/min}}{5\ \text{min}}$ → $\boxed{-4\ \text{m/min}^2}$

D-C2

4. Suppose that the amount of money m, in thousands of dollars, a business has in cash reserves at time t, in months after the start of the new year is modeled by the differentiable function $m = \frac{\sin(6t)}{t+0.5} + 5$. Explain the meaning of $m'(3) = 1.193$, including specific discussion on units.

$\frac{dm}{dt}$ ← \$1,000 / ← Months

t, months

When it is 3 months after the new year, the business' cash reserves are growing at a rate of \$1193 per month.

5. The number b of bacteria cells in a culture t days after an experiment has started can be modeled by the differentiable function $b(t) = 600e^{-0.5t}$. Explain the meaning of $b(3) = 133.878$ using specific units. Then explain the meaning of $\frac{db}{dt}|_{t=3} = -66.939$ using specific units.

$\frac{db}{dt}$ ← cells / ← day → unit: cells/day

Three days ~~have~~ after an experiment has started, the culture is decreasing at a rate of -66.939 cells per day.

*(Can leave off the negative if you use the word decreasing)

D-CD7

6. Write the equation of the line tangent to $y = (2x-1)^4$ where $x=\underline{1}_{x_1}$

y_1: $y(1) = (2(1)-1)^4 = (1)^4 = \underline{1}$ ← y_1

$y - y_1 = m(x - x_1)$

$y - 1 = m(x-1)$

$m = \frac{dy}{dx}\Big|_{x=1}$

$\frac{dy}{dx} = 4(2x-1)^3 \cdot 2 \rightarrow 8(2x-1)^3$

plug in x=1 → $8(2\cdot1-1)^3 = \underline{\underline{8}}_m$

$\boxed{y - 1 = 8(x-1)}$

7. Write the equation of a line with a slope of 6 that is tangent to $y = x^2 - 4x + 3$.

given m! Need x_1 and y_1.... hmm...

$y - y_1 = m(x-x_1)$

Idea: find $\frac{dy}{dx}$, set = 6, solve for x!

$y - y_1 = 6(x - x_1)$
8 5

$\frac{dy}{dx} = 2x - 4 = 6$

$2x = 10$

$\underline{x = 5}$ ← x_1 !

Find y_1 ✓

$y(5) = 5^2 - 4(5) + 3 = 25 - 20 + 3 = 8$

$y - 8 = 6(x-5)$

Is f continuous @ $x=1$?
Is f' continuous @ $x=1$? $\xrightarrow{\text{Both yes}}$ f differentiable

D-CD4

8. Show that $f(x) = \begin{cases} 5x^2 - 3x - 6 & x \le 1 \\ -2x^2 - 2 & x > 1 \end{cases}$ is not differentiable at $x = 1$.

Is f continuous @ $x=1$?

$\lim_{x\to1^-} f(x) = f(1) = \lim_{x\to1^+} f(x)$

$5-3-6 = 5-3-6 = -2-2$

$-4 = -4 = -4$

✓ yes

$f'(x) = \begin{cases} 10x-3, & x \le 1 \\ -4x, & x>1 \end{cases}$

$\lim_{x\to1^-} f'(x) = f'(1) = \lim_{x\to1^+} f'(x)$

$10(1) - 3 = 10(1) - 3 = -4(1)$

$7 = 7 \ne -4$ ★ !

f is cont. @ $x=1$
f' is not.
So f is not differentiable @ $x=1$.

9. Find the values of a and b that would make $f(x)$ differentiable. $f(x) = \begin{cases} ax^2 + bx - 2 & x \le 2 \\ -2x^2 + 2x + 8 & x > 2 \end{cases}$

Make f continuous.

$\lim_{x\to2^-} f(x) = f(2) = \lim_{x\to2^+} f(x)$

$4a + 2b - 2 = -2\cdot2^2 + 2\cdot2 + 8$
$\qquad -8+4+8$

$4a + 2b - 2 = 4$

$4a + 2b = 6$

Make f' continuous

$f'(x) = \begin{cases} 2ax + b, & x \le 2 \\ -4x + 2, & x > 2 \end{cases}$

$\lim_{x\to2^-} f'(x) = f'(2) = \lim_{x\to2^+} f'(x)$

$4a + b = -6 \quad \leftarrow -4(2)+2$

$\begin{cases} 4a + b = -6 \\ -(4a + 2b = 6) \end{cases}$

$0a - 1b = -12$

$\boxed{b = 12}$

$4a + 12 = -6$

$4a = -18$

$\boxed{a = -18/4 = -9/2}$

D-AD18

10. Use a tangent line to approximate the value of $\sqrt[3]{29}$

General function: $y = \sqrt[3]{x} = x^{1/3}$

Close value: $\sqrt[3]{27} = 3 \rightarrow (27, 3)$

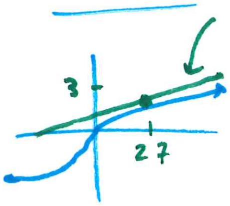
Tan line: $y - 3 = m(x - 27)$

$y - 3 = \frac{1}{27}(x - 27)$

Plug in $x = 29$

$y - 3 = \frac{1}{27}(2)$

$y = 3\frac{2}{27}$



need m: $\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$

$\frac{dy}{dx} = \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}} \xrightarrow[\text{in } 27]{\text{plug}} \frac{1}{3\sqrt[3]{27^2}} = \frac{1}{3\sqrt[3]{729}} = \frac{1}{3\cdot9} = \frac{1}{27}$

m

$\begin{array}{r} 27 \\ \times 27 \\ \hline 729 \end{array}$

★11. Use a tangent line to approximate the value of $\cos\frac{\pi}{5}$ [Note: $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$]

(Won't be on test)

$\frac{dy}{dx} = -\sin(x)$

@ $x = \pi/6$

$\left.\frac{dy}{dx}\right|_{\pi/6} = -\sin(\pi/6)$

$= -1/2$

$y - \frac{\sqrt{3}}{2} = m\left(x - \frac{\pi}{6}\right)$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(x - \frac{\pi}{6}\right)$

Plug in $\pi/5$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{\pi}{5} - \frac{\pi}{6}\right)$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{6\pi}{30} - \frac{5\pi}{30}\right)$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{\pi}{30}\right)$

$y - \frac{\sqrt{3}}{2} = -\frac{\pi}{60}$

$\boxed{y = \frac{\sqrt{3}}{2} - \frac{\pi}{60}}$

Set numerator of derivative $=0$, solve

Set denominator of deriv $=0$, solve

D-CD5

12. Find the x-values of any horizontal and vertical tangents to $f(x) = x^4 - 8x^2 + 2$.

$f'(x) = 4x^3 - 16x$

V.T. No denom... So, none!

H.T. $4x^3 - 16x = 0$

$4x(x^2-4) = 0$

$4x(x+2)(x-2) = 0$

$\boxed{x=0 \quad x=-2 \quad x=2}$ H.T.

13. Find the x-values of any horizontal and vertical tangents to $y = 2\sqrt{x} - \frac{1}{2}x^2$ $= 2x^{1/2} - \frac{1}{2}x^2$

$\frac{dy}{dx} = 1 \cdot x^{-1/2} - x^1$

factor out $x^{-1/2}$

$\frac{dy}{dx} = x^{-1/2}\left(1 - x^{3/2}\right)$

$\frac{dy}{dx} = \frac{1 - x^{3/2}}{x^{1/2}}$

H.T. $1 - x^{3/2} = 0$

$1 = x^{3/2}$) Square both sides

$1 = x^3$

$\boxed{1 = x}$ H.T.

V.T. $\sqrt{x} = 0$

$x = 0$

D-AD0

Evaluate each limit using L'hopital's Rule.

14. $\lim_{\theta\to 0} \frac{\sin 8\theta}{6\theta} = \frac{0}{0}$:|

l'hôp

$\lim_{\theta\to 0} \frac{8\cos 8\theta}{6} = \frac{8\cos(8\cdot 0)}{6} = \frac{8}{6} = \frac{4}{3}$

15. $\lim_{x\to 0} \frac{e^{3x} - 2^x}{3x} = \frac{1-1}{0} = \frac{0}{0}$:-|

l'hôp

$\lim_{x\to 0} \frac{3e^{3x} - 2^x \cdot \ln 2}{3} = \frac{3e^0 - 2^0 \cdot \ln 2}{3}$

$\frac{3 - \ln 2}{3}$

$n^0 = 1$, for all $n \neq 0$

16. $\lim_{x\to\infty} x^2 e^{-x}$??

rewrite

l'hôp

$\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$:(

$\lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$:(

l'hôp again!

$\lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$

Answer

D-AD4

Evaluate each.

17. $f(x) = \sqrt{2x^2 + 4}$. Find $f'(4)$

See next Page

18. If $y = \cot^2 5t$, then what is $\frac{dy}{dt}$?

D-AD 4

17. $f(x) = \sqrt{2x^2+4}$ $\quad f'(4)$?

$f(x) = (2x^2+4)^{1/2}$

$f'(x) = \frac{1}{2}(2x^2+4)^{-1/2} \cdot 4x$

$f'(x) = 2x\,(2x^2+4)^{-1/2}$

$f'(x) = \dfrac{2x}{(2x^2+4)^{1/2}}$

$f'(x) = \dfrac{2x}{\sqrt{2x^2+4}} \longrightarrow f'(4) = \dfrac{2\cdot 4}{\sqrt{2\cdot 4^2+4}} = \dfrac{8}{\sqrt{36}} = \dfrac{8}{6} \rightarrow \dfrac{4}{3}$

18. $y = \cot^2(5t)$ $\quad \frac{dy}{dt}$?

$y = [\cot(5t)]^2$

$\frac{dy}{dt} = 2[\cot(5t)]^1 \cdot -\csc^2(5t) \cdot 5$

$\boxed{-10\cot(5t)\csc^2(5t)}$