2.8 The RULES: Power Product Quotient Chain

447. Let $f(x) = \begin{cases} 3-x & x<1 \\ ax^2+bx & x \geq 1 \end{cases}$ where a and b are constants.

a) If the function is continuous for all x, what is the relationship between a and b?

b) Find the unique values for a and b that will make f both continuous and differentiable.

448. Suppose that $u(x)$ and $v(x)$ are differentiable functions of x and that

$$u(1) = 2, \qquad u'(1) = 0, \qquad v(1) = 5, \qquad \text{and} \qquad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

a) $\dfrac{d}{dx}(uv)$ b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ d) $\dfrac{d}{dx}(7v-2u)$

449. Graph the function $y = \dfrac{4x}{x^2+1}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called *Newton's serpentine.*) Find the tangent lines at the origin and at the point $(1, 2)$.

450. Graph the function $y = \dfrac{8}{x^2+4}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called the *witch of Agnesi.*) Find the tangent line at the point $(2, 1)$.

FIND THE DERIVATIVE OF THE GIVEN FUNCTION. EXPRESS YOUR ANSWER IN SIMPLEST FACTORED FORM.

451. $A(z) = (3z-5)^4$

452. $q(u) = (3u^5 - 2u^3 - 3u - \frac{1}{3})^3$

453. $b(y) = (y^3-5)^{-4}$

454. $c(d) = \sqrt[3]{(5d^2-1)^5}$

455. $u(p) = \dfrac{3p^2-5}{p^3+2p-6}$

456. $V(x) = \dfrac{\sqrt{5x^3}}{5x^3}$

457. $f(x) = 3x^{1/3} - 5x^{-1/3}$

458. $g(z) = \dfrac{1}{\sqrt{36-z^2}}$

459. $p(t) = (3-2t)^{-1/2}$

460. $h(u) = \sqrt{u-1}\sqrt[3]{2u+3}$

461. $f(x) = \dfrac{3x}{x+5}$

462. $g(y) = \dfrac{4y-3}{3-2y}$

463. $p(x) = \dfrac{x^2+10x+25}{x^2-10x+25}$

464. $m(x) = \dfrac{7x}{1-3x}$

465. $f(x) = \dfrac{3}{x^2} - \dfrac{x^2}{3}$

466. $g(x) = \left(\dfrac{4x-3}{5-3x}\right)(2x+7)$

467. $F(x) = 10x^{27} - 25x^{1/5} + 12x^{-12} + 350$

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction. —*Leo Tolstoy*