2.8 The RULES: Power Product Quotient Chain

447. Let $f(x) = \begin{cases} 3-x & x<1 \\ ax^2+bx & x \geq 1 \end{cases}$ where a and b are constants.

a) If the function is continuous for all x, what is the relationship between a and b?

b) Find the unique values for a and b that will make f both continuous and differentiable.

448. Suppose that $u(x)$ and $v(x)$ are differentiable functions of x and that

$$u(1) = 2, \qquad u'(1) = 0, \qquad v(1) = 5, \qquad \text{and} \qquad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

a) $\dfrac{d}{dx}(uv)$ b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ d) $\dfrac{d}{dx}(7v-2u)$

449. Graph the function $y = \dfrac{4x}{x^2+1}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called *Newton's serpentine.*) Find the tangent lines at the origin and at the point $(1, 2)$.

450. Graph the function $y = \dfrac{8}{x^2+4}$ on your calculator in the window $-5 \leq x \leq 5$, $-3 \leq y \leq 3$. (This graph is called the *witch of Agnesi.*) Find the tangent line at the point $(2, 1)$.

Find the derivative of the given function. Express your answer in simplest factored form.

451. $A(z) = (3z-5)^4$

452. $q(u) = (3u^5 - 2u^3 - 3u - \frac{1}{3})^3$

453. $b(y) = (y^3-5)^{-4}$

454. $c(d) = \sqrt[3]{(5d^2-1)^5}$

455. $u(p) = \dfrac{3p^2-5}{p^3+2p-6}$

456. $V(x) = \dfrac{\sqrt{5x^3}}{5x^3}$

457. $f(x) = 3x^{1/3} - 5x^{-1/3}$

458. $g(z) = \dfrac{1}{\sqrt{36-z^2}}$

459. $p(t) = (3-2t)^{-1/2}$

460. $h(u) = \sqrt{u-1}\,\sqrt[3]{2u+3}$

461. $f(x) = \dfrac{3x}{x+5}$

462. $g(y) = \dfrac{4y-3}{3-2y}$

463. $p(x) = \dfrac{x^2+10x+25}{x^2-10x+25}$

464. $m(x) = \dfrac{7x}{1-3x}$

465. $f(x) = \dfrac{3}{x^2} - \dfrac{x^2}{3}$

466. $g(x) = \left(\dfrac{4x-3}{5-3x}\right)(2x+7)$

467. $F(x) = 10x^{27} - 25x^{1/5} + 12x^{-12} + 350$

A man is like a fraction whose numerator is what he is and whose denominator is what he thinks of himself. The larger the denominator, the smaller the fraction. —*Leo Tolstoy*

2.9 Trigonometric Derivatives

FIND $\dfrac{dy}{dx}$ FOR EACH OF THE FOLLOWING.

468. $y = 3\cos x$

469. $y = \cot x$

470. $y = \tan x - x$

471. $y = x\sin x + \cos x$

472. $y = \sin\left(\frac{3\pi x}{2}\right)$

473. $y = \cos^2 x$

474. $y = \tan^3 x$

475. $y = \sin\sqrt{x}$

476. $y = \cos(3x+1)$

477. $y = \sin^2(4x)$

478. $y = 2\sin x\cos x$

479. $y = \pi\cot(\pi x)$

480. $y = x^2\tan x$

481. $y = 8\csc 8x$

482. Find all points on the curve $y = \tan x$ over the interval $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ where the tangent line is parallel to the line $y = 2x$.

483. Graph $y = 1 + \sqrt{2}\csc x + \cot x$ on your calculator in the window $0 \le x \le \pi$, $-1 \le y \le 9$. Find the equation of the tangent line at the point $\left(\frac{\pi}{4}, 4\right)$; then find the point on the graph where the graph has a horizontal tangent.

484. Is there a value of b that will make $g(x) = \begin{cases} x+b & x<0 \\ \cos x & x \ge 0 \end{cases}$ continuous at $x=0$? Differentiable at $x=0$? Justify your answers.

485. Find the 1000th derivative of $\cos x$.

486. Find the tangent to the curve $y = 2\tan\left(\frac{\pi x}{4}\right)$ at $x = 1$.

FIND y'' FOR EACH OF THE FOLLOWING.

487. $y = \csc\theta$

488. $y = \sec\theta$

489. $y = 2 - 2\sin\theta$

490. $y = \sin\theta + \cos\theta$

Neither in the subjective nor in the objective world can we find a criterion for the reality of the number concept, because the first contains no such concept, and the second contains nothing that is free from the concept. How then can we arrive at a criterion? Not by evidence, for the dice of evidence are loaded. Not by logic, for logic has no existence independent of mathematics: it is only one phase of this multiplied necessity that we call mathematics. How then shall mathematical concepts be judged? They shall not be judged. Mathematics is the supreme arbiter. From its decisions there is no appeal. We cannot change the rules of the game, we cannot ascertain whether the game is fair. We can only study the player at his game; not, however, with the detached attitude of a bystander, for we are watching our own minds at play. —*Dantzig*