475. $y = \sin\left(x^{1/2}\right)$

$$\frac{dy}{dx} = \cos\left(x^{1/2}\right) \cdot \frac{1}{2} x^{-1/2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cdot \cos(\sqrt{x})$$

$$\boxed{\frac{dy}{dx} = \frac{\cos(\sqrt{x})}{2\sqrt{x}}}$$

476. $y = \cos(3x+1)$

$$y' = -\sin(3x+1) \cdot 3$$

$$\underline{y' = -3\sin(3x+1)}$$

477. $y = \sin^2(4x) = (\sin(4x))^2$

$y' = 2(\sin(4x))' \cos(4x) \cdot 4$

$$\boxed{y' = 8\sin(4x)\cos(4x)}$$

CAN GO FARTHER... see $\sin(\alpha+\beta)$

$$8 \cdot \left(\frac{1}{2}\sin(8x)\right)$$

$$y' = 4\sin(8x)$$

478. $y = \underbrace{2\sin(x)}_{f} \cdot \underbrace{\cos(x)}_{g}$

$f' = 2\cos(x)$

$g' = -\sin(x)$

$y' = f'g + fg'$

$2\cos(x) \cdot \cos(x) + 2\sin(x) \cdot -\sin(x)$

$2\cos^2(x) - 2\sin^2(x)$

$2(\cos^2(x) - \sin^2(x))$

479. $y = \pi \cdot \cot(\pi x)$

$y' = \pi \cdot -\csc^2(\pi x) \cdot \pi$

$y' = -\pi^2 \cdot \csc^2(\pi x)$

or

$-(\pi \csc(\pi x))^2$

480. $y = \underset{f}{\underline{x^2}} \cdot \underset{g}{\underline{\tan(x)}}$ $\quad f' = 2x$, $\quad g' = \sec^2(x)$

$f'g + fg'$

$\frac{dy}{dx} = 2x\tan(x) + x^2 \cdot \sec^2(x)$

$= x(2\tan(x) + x \cdot \sec^2(x))$

481. $y = 8\csc(8x)$

$y' = -8\csc(8x)\cot(8x) \cdot 8$

$\underline{y' = -64\csc(8x)\cot(8x)}$

482. Parallel to $y=2x$

means slope is 2

slope $\Rightarrow$ derivative

$y = \tan(x)$

$\frac{dy}{dx} = \sec^2(x) = 2$

$\sec(x) = \pm\sqrt{2}$

take reciprocal of both sides

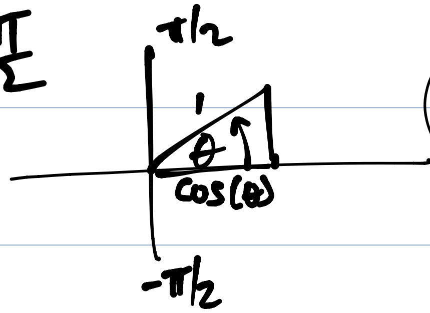
$\frac{1}{\cos(x)} = \pm\sqrt{2}$

$\cos(x) = \pm\frac{1}{\sqrt{2}}$

482 cont'd.

$$\cos(x) = \frac{\pm\sqrt{2}}{2}$$

$$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$$



$$x = \frac{\pi}{4}, -\frac{\pi}{4}$$

485.

	n^{th} Derivative
$y = \cos(x)$	0
$y' = -\sin(x)$	1
$y'' = -\cos(x)$	2
$y'''(x) = \sin(x)$	3
$y^{(4)}(x) = \cos(x)$	

$$\frac{1000}{4} = 250 \text{ (remainder } 0)$$

go to this step

$y^{(1000)} = y = \cos(x)$

488. $y = \sec(\theta)$

$y' = \underbrace{\sec(\theta)}_{f}\underbrace{\tan(\theta)}_{g}$

$f' = \sec(\theta)\tan(\theta) \quad g' = \sec^2(\theta)$

$f'g + fg'$

$y'' = \overbrace{\sec(\theta)\tan(\theta)}^{f'} \cdot \overbrace{\tan(\theta)}^{g} + \overbrace{\sec(\theta)}^{f} \cdot \overbrace{\sec^2(\theta)}^{g'}$

$y'' = \sec(\theta)\tan^2(\theta) + \sec^3(\theta)$

$y'' = \sec(\theta)\left(\tan^2(\theta) + \sec^2(\theta)\right)$

447. $f(x): \begin{cases} 3-x, & x<1 \\ ax^2+bx, & x \geq 1 \end{cases}$

a) Continuity: $\lim_{x \to 1^-} f = f(1) = \lim_{x \to 1^+} f(x)$

$$3-1 = a(1^2)+b(1)$$

$$2 = a+b$$

b) Take derivative

$$f'(x) = \begin{cases} -1, & x<1 \\ 2ax+b, & x \geq 1 \end{cases}$$

If f is differentiable, f' is continuous.

$$\lim_{x \to 1^-} f'(x) = f'(1) = \lim_{x \to 1^+} f'(x)$$

$$-1 = 2a+b \stackrel{\checkmark}{=} 2a+b$$

$$-1 = 2a+b$$

return to part a.

$$\begin{cases} 2 = a + b \\ -1 = 2a + b \end{cases}$$ ← Solve system

<u>Elimination</u>

$$\begin{cases} 2 = a + b \\ -(-1 = 2a + b) \end{cases}$$

$$3 = -a + \cancel{0b}$$

$$-3 = a$$

$$2 = a + b$$

$$2 = -3 + b$$

$$5 = b$$

448.

a) $u'v + uv'$

$u'(1)v(1) + u(1)v'(1)$

$0 \cdot 5 + 2 \cdot -1$

$0 + -2$

-2

b) $\frac{u'v - uv'}{v^2}$

$\frac{\cancel{0 \cdot 5} - 2 \cdot -1}{5^2}$

$\frac{2}{25}$

c) $\frac{v'u - vu'}{u^2}$

$\frac{-1 \cdot 2 - 5 \cdot 0}{2^2}$

$\frac{-2}{4} = \frac{-1}{2}$

d) $\frac{d}{dx}(7v - 2u)$

$7 \cdot v'(x) - 2u'(x)$

$7(5) - 2(0)$

35

455. $$u(p) = \frac{3p^2-5 \leftarrow f}{p^3+2p-6 \leftarrow g}$$

$$f' = 6p \qquad g' = 3p^2+2$$

$$\frac{f'g - fg'}{g^2}$$

$$u'(p) = \frac{6p\,(p^3+2p-6) - (3p^2-5)(3p^2+2)}{(p^3+2p+6)^2}$$

numer-
ator → $6p^4+12p^2-36p-(9p^4-9p^2-10)$

$$6p^4+12p^2-36p-9p^4+9p^2+10$$

$$\boxed{u'(p) = \frac{-3p^4+21p^2-36p+10}{(p^3+2p+6)}}$$

459. $p(t) = (3-2t)^{-1/2}$

$$p'(t) = -\cancel{\frac{1}{2}}(3-2t)^{-3/2} \cdot -\cancel{2}$$

$$p'(t) \quad (3-2t)^{-3/2}$$ or

$$p'(t) = \frac{1}{\sqrt{(3-2t)^3}}$$

464. $m(x) : \dfrac{7x \leftarrow f}{1-3x \leftarrow g}$

$f' = 7 \quad g' = -3$

$\dfrac{f'g - fg'}{g^2} \quad \dfrac{7(1-3x) - (7x)(-3)}{(1-3x)^2}$

$$m'(x) = \frac{7 \cancel{-21x} + \cancel{21x}}{(1-3x)^2}$$

$$\boxed{m'(x) : \frac{7}{(1-3x)^2}}$$