Good afternoon and welcome back: warm up

Next assessment: Monday

If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) -2 (B) 0 (C) 2 (D) 4 (E)

(E) not defined

$6x + 2y + 2xy' + 2yy' = 0$

$y'(2x+2y) = -6x-2y$

$y' = \frac{-6x-2y}{2x+2y}$

Now plug in x=1...but what's y?

That's what the original function tells us!

Plug x=1 into that

$3(1)^2+2(1)(y)+y^2=2$

$3+2y+y^2=2$

$y^2+2y+1=0$

$(y+1)(y+1)=0$

$y = -1$

$y' = \frac{-6-2y}{2+2y}$

$y' = \frac{-6-2(-1)}{2+2(-1)}$

$\frac{-4}{0}$

More Practice with Implicit Differentiation

The slope of the line tangent to the curve $y^2+(xy+1)^3=0$ at $(2,-1)$ is

chain rule

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

distribute

$2y\cdot y' + 3(xy+1)^2(1y + xy') = 0$

$2yy' + 3(xy+1)^2 y + 3(xy+1)^2 \cdot xy' = 0$

$2(x+y)$

$2x+2y$

$2yy' + 3(xy+1)^2 xy' = -3(xy+1)^2 \cdot y$

$y'(2y+3x(xy+1)^2) = -3(xy+1)^2 \cdot y$

$y' = \dfrac{-3(xy+1)^2 y}{2y+3x(xy+1)^2}$

$(2,-1)$

$3/4$

The second derivative by implicit differentiation

$y^2 - xy = 8$ (prod rule) f: −x g: y Find y''

$2yy' - 1 \cdot y + -x \cdot y' = 0$ f: −x g: y; f': −1 g': y'

$2yy' - y - xy' = 0$

$\frac{d}{dx} y' = \frac{d}{dx} \frac{y}{2y - x}$ f

$\frac{f'g - fg'}{g^2}$ f: y f': y'; g: 2y − x g': 2y' − 1

$y'' = \frac{y' \cdot (2y - x) - y \cdot (2y' - 1)}{(2y - x)^2}$ ← quot. rule

$\frac{2yy' - xy' - 2yy' + y}{(2y - x)^2}$ Dist.

$y'' = \frac{-xy' + y}{(2y - x)^2}$ recall: $y' = \frac{y}{2y - x}$

$y'' \; \frac{-x \cdot \frac{y}{2y - x} + y}{(2y - x)^2}$ whew ☺

How to Simplify further: multiply by the compound denominator

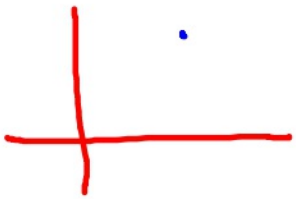
$\frac{-x \cdot \frac{y}{2y - x} + y}{(2y - x)^2} \cdot \frac{2y - x}{2y - x}$ "1"

$\frac{-xy + y(2y - x)}{(2y - x)^3} \rightarrow \frac{-xy + 2y^2 - xy}{(2y - x)^3}$

$\frac{2y^2 - 2xy}{(2y - x)^3}$

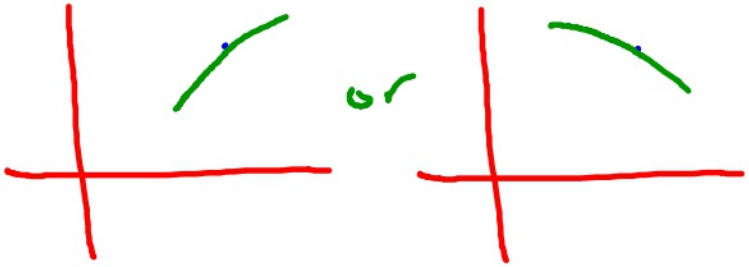
What does y tell you?

where to go, a point in space for a particular x



What does y' tell you?

increasing or decreasing as you pass through that point



What does y'' tell you?

increasingly or decreasingly increasing or decreasing

For $2x^3-3y^2=8$, show that $y'' = \frac{2xy^2-x^4}{y^3}$

Warm up
Tues.

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

HW due Thursday
"Misc. Review" handout