AP Calculus

Assessments are being passed back

Turn review packets in if not yet turned in

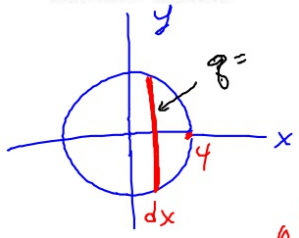
Hopefully you watched the videos yesterday...(if not, do so tonight!!)

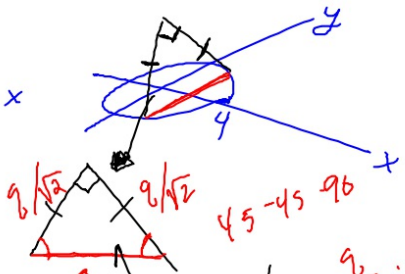
Today's goals: volumes of cross sections practice + slope fields
(need to have sheet I handed out last week)

5. Find the volume of a solid **S** if its base is bounded by the circle $x^2 + y^2 = 16$ and the cross sections perpendicular to the x- axis are isosceles right triangles having the hypotenuse in the plane of the base.

Notes

1. Graph region
2. Draw rep. rectangle, 3D view, flat view.
3. Write Basic Integral.
4. Be sure A(x) is in terms of x.

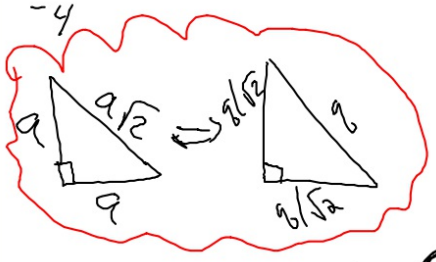


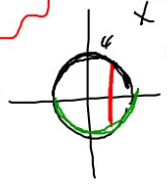


$\int_{-4}^{4} A(x)\,dx$

$A(x) = \frac{1}{2} \cdot \frac{q}{\sqrt{2}} \cdot \frac{q}{\sqrt{2}}$

$A(x) = \frac{1}{4} q^2$





$q = \sqrt{16-x^2} - -\sqrt{16-x^2}$

$q = 2\sqrt{16-x^2}$

"top - bottom"

$A(x) = \frac{1}{4}\left(2\sqrt{16-x^2}\right)^2$

$\int_{-4}^{4} \frac{1}{4}\left(2\sqrt{16-x^2}\right)^2 dx$

$\leftarrow \int_{-4}^{4} 16 - x^2\,dx.$

$\frac{256}{3}$

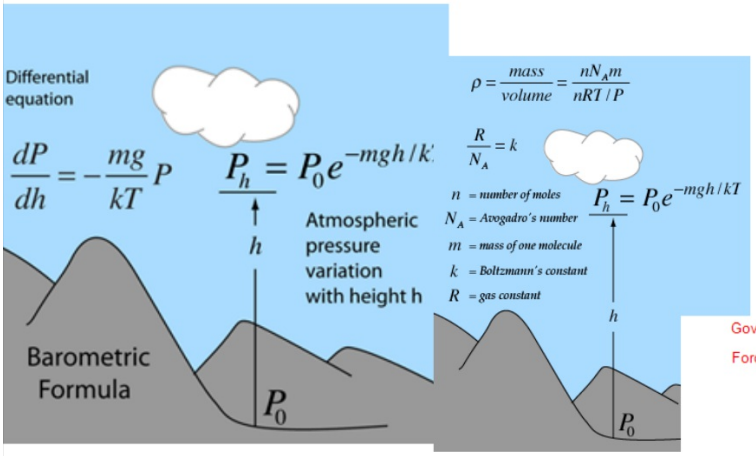
Differential Equations:

What are they? → an equation that involves a derivative.

What do they do?

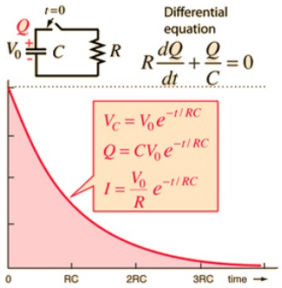
How do I use them?

What are differential equations good for?



atmospheric pressure

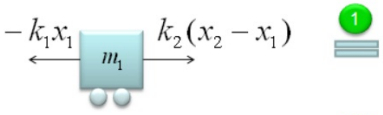
electrical circuits



force and mass

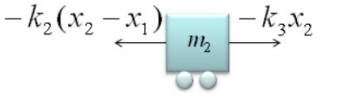
Governing rule :

Forces Acting on Each Mass



1

$m_1 \frac{d^2x_1}{dt^2}$

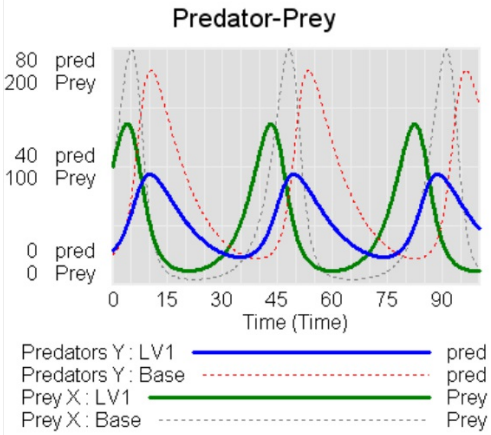


2

$m_2 \frac{d^2x_2}{dt^2}$

Motion of Each Mass

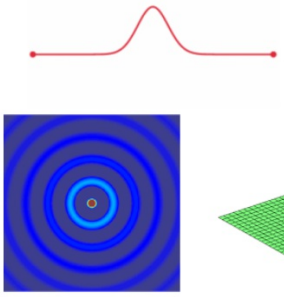
Population/Competition



$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

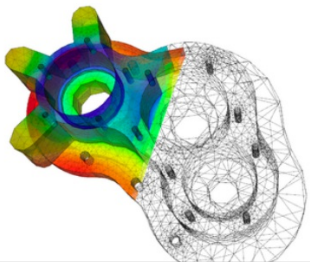
Wave Dynamics



$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Heat

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$



Differential equations	Meaning
$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	The electric field leaving a volume is proportional to the charge inside.
$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
$\nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial \mathbf{E}}{\partial t}\right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Basic questions:

Bacteria in a culture are observed to grow at a rate proportional to the number of cells present. At the beginning of an experiment, there are 10,000 cells present. After 3 hours, there are 500,000. How many will there be after 24 hours? What is the doubling time for this system?

$$y = e^{3x}$$

$$\frac{dy}{dx} = 3\boxed{e^{3x}}$$

$$\frac{dy}{dx} = 3y.$$

Slope Fields: A way to visualize solutions to differential equations

- Very easy to make!!!!
- Just plug coordinates of a point into dy/dx, simplify to a number, then sketch a little line segment/gradient with slope equal to that number!