AP Calculus

Differential Equations: recognizing, translating, and separating

Warm up (please do in notes) *Solve each for x:*

1.) $2\ln(x) - 3 = 7$

$\log_e = \ln$

$\ln(x) = 5$

$e^5 = x$

2.) $e^{x+2} = 6$

$\ln e^{x+2} = \ln 6$

$\log_e e^{x+2}$

$x+2 = \ln 6$

$\boxed{x = \ln 6 - 2}$

logarithms <u>are</u> exponents.

Important Log/Exponent Properties to know: $\log_{10} 1{,}325{,}604 \approx$

$\log_b a = x \Leftrightarrow b^x = a$

$\ln(ab) = \ln(a) + \ln(b) \approx a^x \cdot a^y = a^{x+y}$ $\quad a * \ln(b) \Leftrightarrow \ln(b^a)$

$\ln(a/b) = \ln(a) - \ln(b)$

What is a diff eq?

ex 1: The rate of change of the volume V of water in a tank with respect to time t is directly proportional to the square root of the volume. Write a differential equation that describes this relationship.

$$\frac{dV}{dt} = K\sqrt{V}$$

$\boxed{K = \text{factor of the proportion}}$

12. The rate of change of the volume, $V$, of water in a tank with respect to time, $t$, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

(C) $\frac{dV}{dt} = k\sqrt{t}$

(D) $\frac{dV}{dt} = \frac{k}{\sqrt{V}}$

(E) $\frac{dV}{dt} = k\sqrt{V}$

Another: Out of a population of N people, a rumor is spreading at a rate proportional to the product of the people who have heard it and the people who haven't. If p is the number of people who have heard it, write a diff eq that models this behavior.

$$\frac{dp}{dt} = K \cdot p \cdot (N - p)$$

Your turn: The amount of bacteria in a culture B is growing at a rate that is proportional to the cube root of the bacteria present.

Write a differential equation that models this behavior:

$$\left[ \frac{dB}{dt} = K \cdot \sqrt[3]{B} \right]$$

## Finding the general solution to a differential equation:

Find $y =$ ____

★Separation of Variables★

$$dx \cdot \frac{dy}{dx} = ky \cdot dx$$

sep. of variables

$$\frac{dy}{y} = \frac{ky \cdot dx}{y}$$

group like variables.

$$\int \frac{dy}{y} = \int k \cdot dx$$

★Integrate both sides.

$$\int \frac{1}{y}\,dy = kx + C$$

$$\ln y + \cancel{C} = kx + C$$

$$\quad -\cancel{C} \qquad -C$$

$$\ln y = kx + C$$

def. of log: "exponentiate"

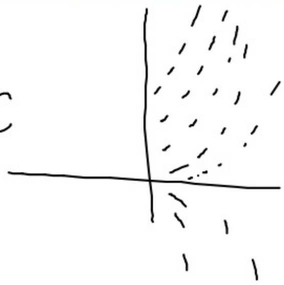
$$e^{kx+C} = y$$

$$y = e^{kx} \cdot \boxed{e^{C}} \Rightarrow \boxed{y = Ce^{kx}}$$

some constant

$$y' = 2x$$

$$y = x^2 + C$$



Compound Interest

$$\underline{y = Pe^{rt}}$$

Shortcut (good to know)

Initial pop.

$$\frac{dy}{dt} = kY \qquad Y = C\,e^{kt}$$

$= 0.05(\$1000)$

$= \$50$

if $\frac{dy}{dx} = \frac{1}{2}y$

then....

$y = Ce^{\frac{1}{2}x}$

84. Population $y$ grows according to the equation $\frac{dy}{dt} = ky$, where $k$ is a constant and $t$ is measured in years. If the population doubles every 10 years, then the value of $k$ is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

$$\frac{dy}{dt} = ky \implies y = Ce^{kt}$$

$$y(0) = Ce^{k \cdot 0}$$
$$= C$$

Init. Pop.

$(0, C)$

$(10, 2C)$

plug into formula

$$y(10) = Ce^{10k}$$

$$\frac{2\cancel{C}}{C} = \frac{\cancel{C}e^{10k}}{\cancel{C}}$$

$$\ln 2 = \ln e^{10k}$$

$$\frac{\ln 2}{10} = \frac{\cancel{10}k}{\cancel{10}}$$

$$A \approx \frac{\ln 2}{10} = k$$

Separation of Variables: Find y if

$$\frac{dy}{dx} = (3y+6)\, dx$$

$$\frac{dy}{(3y+6)} = \frac{\cancel{(3y+6)}\, dx}{\cancel{(3y+6)}}$$

$$\frac{1}{3}\int \frac{3}{3y+6}\, dy = \int dx$$

$$\frac{1}{3}\int \frac{3}{3y+6}\, dy \qquad \frac{1}{3}\ln|3y+6| + C$$

$$= x + C$$

$$\frac{1}{3}\ln|3y+6| + C = x + C$$

$$3\left(\frac{1}{3}\ln|3y+6|\right) = (x + C)\,3$$

$$\ln|3y+6| = 3x + C$$

$$e^{3x+C} = 3y+6$$

$$Ce^{3x} = 3y+6$$

$$Ce^{3x} = 3y+6$$

$$\frac{Ce^{3x}}{3} - \frac{6}{3} = \frac{3y}{3}$$

$$\boxed{Ce^{3x} - 2 = y}$$

Newton's Law of Cooling:

$$\frac{dT}{dt} = k(T-S)$$

where S is the surrounding temperature

Suppose an object that is 1200 C is put into an environment with a constant 80 C temperature. After an hour, the object's temperature is 950 C. Find the temperature after 5 hours in the environment.