

Calculus Mini Lesson

Analyzing a differential equation/slope field

Finding a general solution

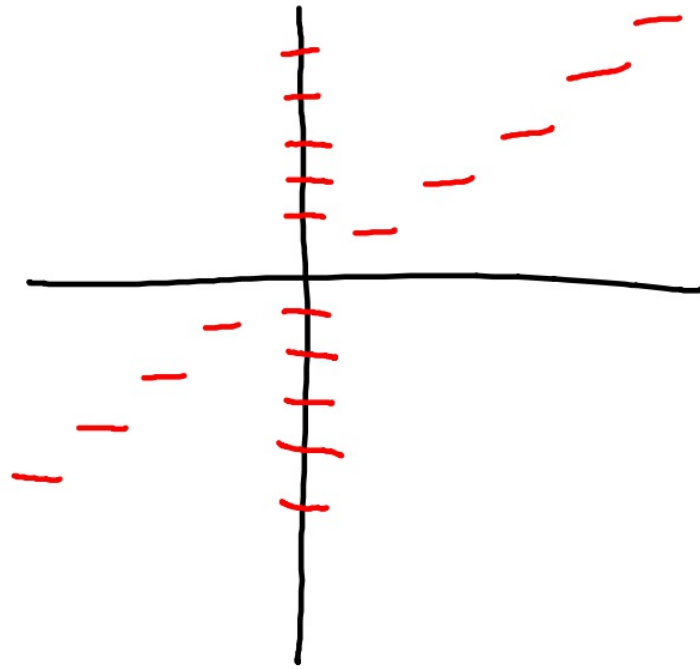
When are the slopes horizontal?

$= 0$

$$\frac{dy}{dx} = x(x-y)$$

• $x = y$

• $x = 0$



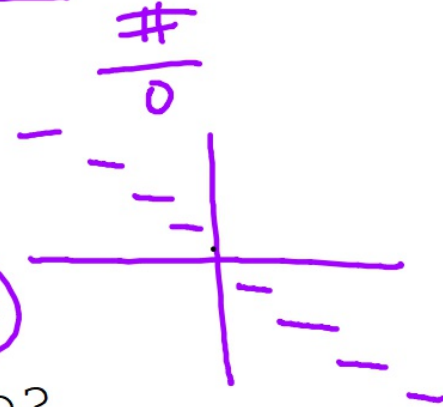
When are the slopes undefined?

$$\frac{dy}{dx} = \frac{x}{x+y} = 0$$

$\frac{0}{0} = \text{indet.}$

$x = -y$
 $x \neq 0$

$y = -x$



When are the slopes positive?

$$\frac{dy}{dx} = -\frac{x^2}{y+3}$$

$-\frac{+}{-}$

$$y' = -\frac{y+3}{x^2}$$

$$-\frac{+}{+}$$

$y < -3$

$\frac{y+3 < 0}{-}$

Find the general solution.

$$y' = 6y - 7$$

$y = \underline{\hspace{2cm}}$

$$\cancel{dx} \left(\frac{dy}{\cancel{dx}} = 6y - 7 \right) dx$$

$$\frac{dy}{6y-7} = \frac{(6y-7) dx}{6y-7}$$

$$\frac{1}{6} \int \frac{1}{6y-7} dy = \int 1 dx$$

$$\frac{1}{6} \int \frac{1}{6y-7} dy = x + C$$

$$\cancel{\frac{1}{6}} \ln|6y-7| = x + C$$

$$\log_e |6y-7| = \underline{6x + C}$$

$$e^{6x+C} = 6y-7$$

$$e^{6x} \cdot e^C = 6y-7$$

$$C e^{6x} = 6y-7$$

$$\frac{C}{6} e^{6x} + \frac{7}{6} = \frac{6y}{6}$$

$$\boxed{C e^{6x} + \frac{7}{6} = y}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$
$$\int \frac{1}{x} dx = \ln|x| + C$$

) solve for y

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$$

Find the general solution.

$$\frac{dy}{dx} = (y-3)2x$$

$$dy = (y-3)2x \cdot dx$$

$$\int \frac{dy}{y-3} = \int 2x \, dx$$

$$\ln|y-3| = x^2 + C$$

$$e^{x^2+C} = y-3$$

$$\boxed{Ce^{x^2} + 3 = y}$$