

Differential Equations:

What are they? equations involving a derivative

What do they do? describe quantities by analyzing their changes

How do I use them? mixing differentiation and integration skills

Differential Equations are "a thing" because it is often easier to study how a quantity changes than to study what that quantity is

What are differential equations good for?

Differential equation

$$\frac{dP}{dh} = -\frac{mg}{kT} P$$

$$P_h = P_0 e^{-mgh/kT}$$

Atmospheric pressure variation with height h

Barometric Formula

atmospheric pressure

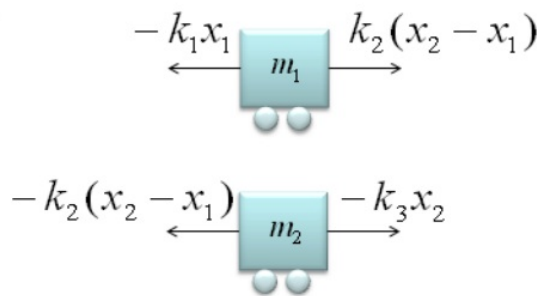
$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{nN_A m}{nRT/P}$$

$$\frac{R}{N_A} = k$$

$$P_h = P_0 e^{-mgh/kT}$$

n = number of moles
 N_A = Avogadro's number
 m = mass of one molecule
 k = Boltzmann's constant
 R = gas constant

Governing rule :
 Forces Acting on Each Mass



electrical circuits

Differential equation

$$R \frac{dQ}{dt} + \frac{Q}{C} = 0$$

$V_C = V_0 e^{-t/RC}$
 $Q = CV_0 e^{-t/RC}$
 $I = \frac{V_0}{R} e^{-t/RC}$

force and mass

Motion of Each Mass

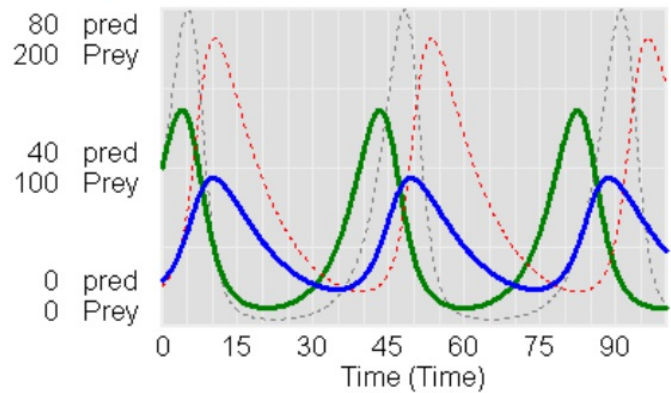
1

$$m_1 \frac{d^2 x_1}{dt^2}$$

2

$$m_2 \frac{d^2 x_2}{dt^2}$$

Population/Competition



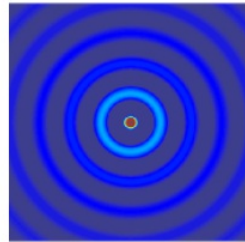
Predators Y: LV1 ———— pred
 Predators Y: Base - - - - - pred
 Prey X: LV1 ———— Prey
 Prey X: Base - - - - - Prey

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

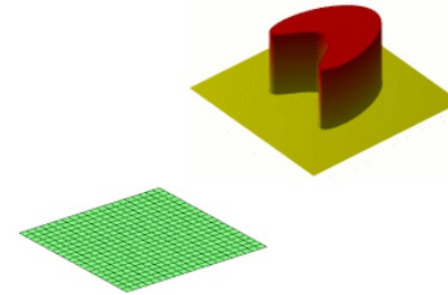
$$y = C e^{kx}$$

Wave Dynamics

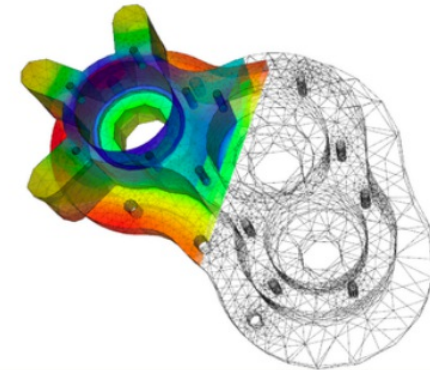


$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Heat



$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$

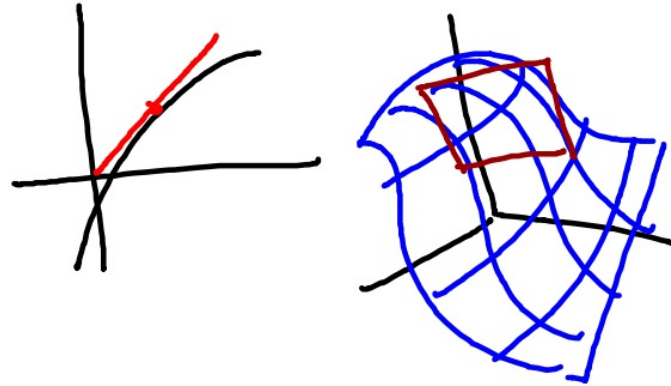




$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$	having a volume is proportional to the charge inside.
$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.



$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$



Basic questions:

this implies exponential growth!!!

Bacteria in a culture are observed to grow at a rate proportional to the number of cells present. At the beginning of an experiment, there are 10,000 cells present. After 3 hours, there are 500,000. How many will there be after 24 hours? What is the doubling time for this system?

$$\star \underbrace{B'}_{\text{growth rate}} = k \cdot B$$
$$B = Ce^{kt}$$

$$y = e^x$$
$$\frac{dy}{dx} = e^x$$

• Direct Variation
 $y = kx$

• Inverse Variation
 $y = \frac{k}{x}$

Slope Fields: A way to visualize solutions to differential equations

Very easy to make!!!!

Just plug coordinates of a point into dy/dx , simplify to a number, then sketch a little line segment/gradient with slope equal to that number!

$$\frac{dy}{dx} = \frac{y+1}{x}$$

Construct the slope field where possible.

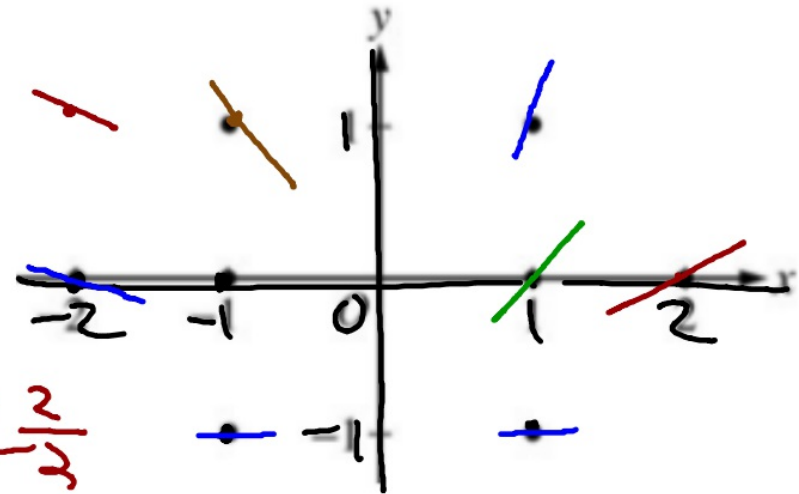
$$(1, 0) \rightarrow \frac{0+1}{1} \rightarrow 1$$

$$(2, 0) \rightarrow \frac{1}{2} \rightarrow \frac{1}{2}$$

$$(0, 0) \rightarrow \frac{1}{0} \rightarrow \text{!!}$$

$$(-1, 1) \rightarrow \frac{2}{-1} \rightarrow -2$$

$$(-2, 1) \rightarrow \frac{2}{-2} \rightarrow -1$$



Describe all points in the plane with a horizontal slope. $y = -1$