Differential Equations:

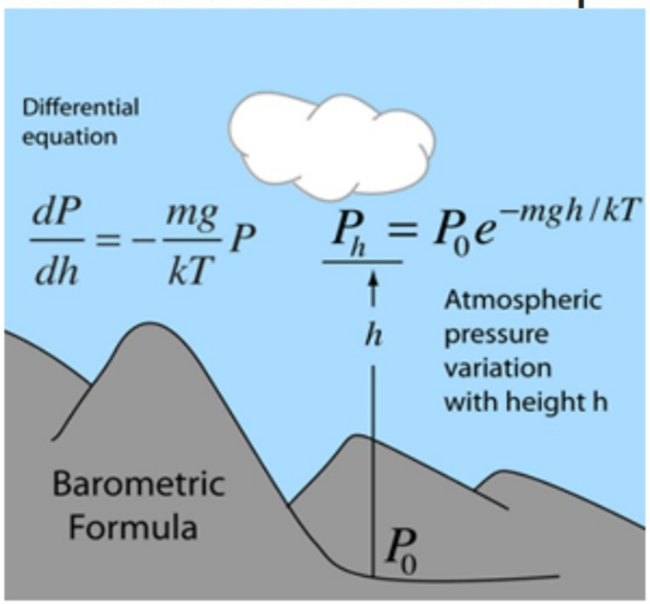
What are they? equations in involving a derivative

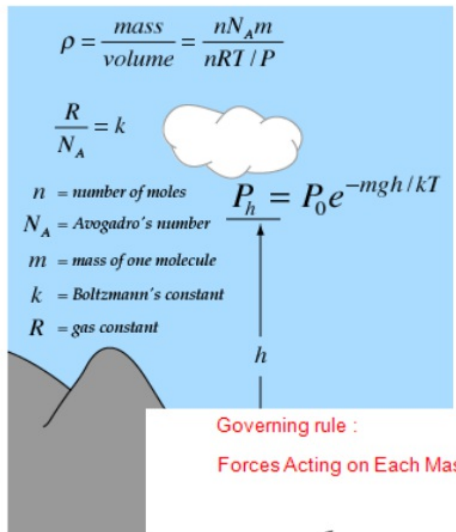
What do they do? describe quantities by analyzing their changes

How do I use them? mixing differentiation and integration skills

Differential Equations are "a thing" because it is often easier to study how a quantity changes than to study what that quantity is

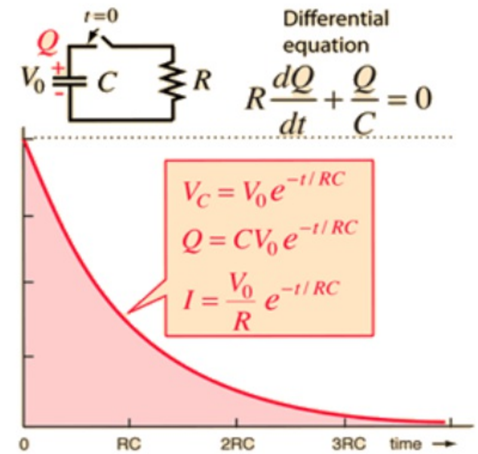
What are differential equations good for?



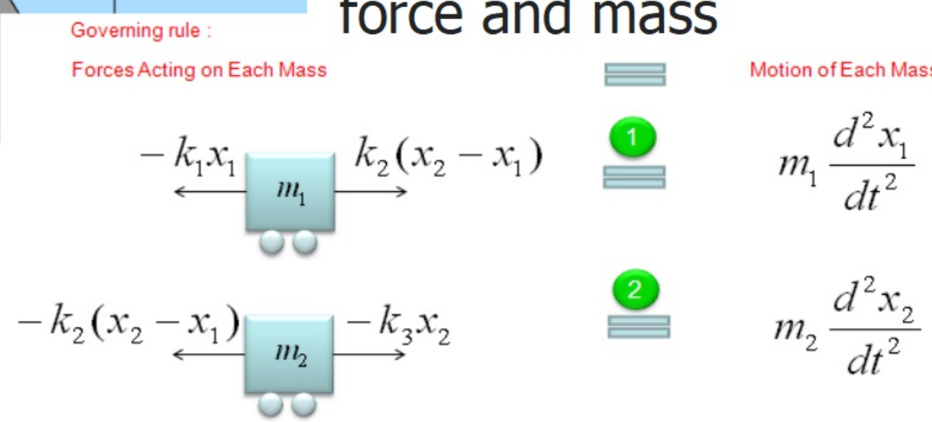


atmospheric pressure

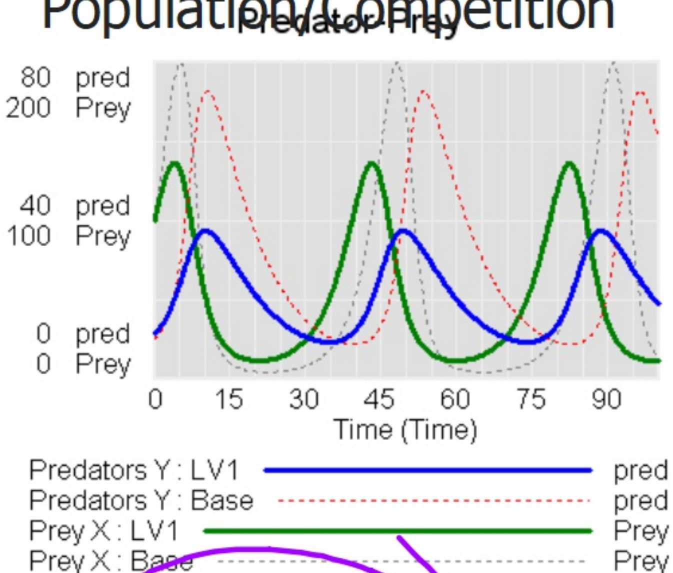
electrical circuits



force and mass



Population/Competition

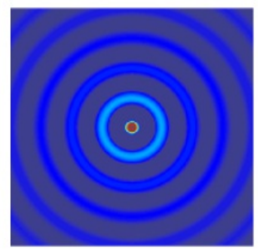


$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \delta xy - \gamma y$$

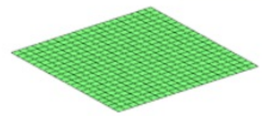
$y = Ce^{kt}$

Wave Dynamics

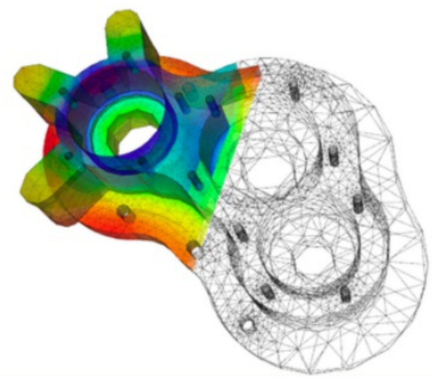


$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

Heat

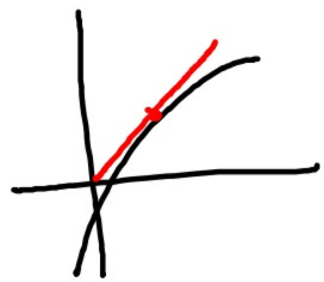


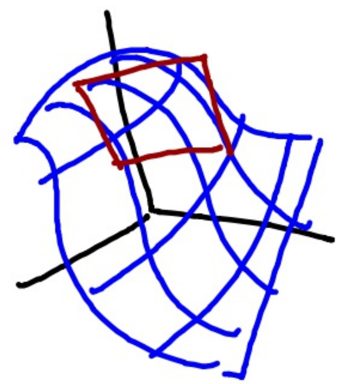
$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$



	ning
ε_0	aving a volume is proportional to the charge inside.
$\nabla \cdot \mathbf{B} = 0$	There are no magnetic monopoles; the total magnetic flux piercing a closed surface is zero.
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	The voltage accumulated around a closed circuit is proportional to the time rate of change of the magnetic flux it encloses.
$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$	Electric currents and changes in electric fields are proportional to the magnetic field circulating about the area they pierce.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$





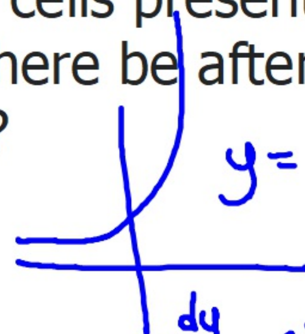
Basic questions:

this implies exponential growth!!!

Bacteria in a culture are observed to grow at a rate proportional to the number of cells present. At the beginning of an experiment, there are 10,000 cells present. After 3 hours, there are 500,000. How many will there be after 24 hours? What is the doubling time for this system?

$\star \; \underbrace{B'}_{\text{growth rate}} = k \cdot B$

$B = Ce^{kt}$



$y = e^x$

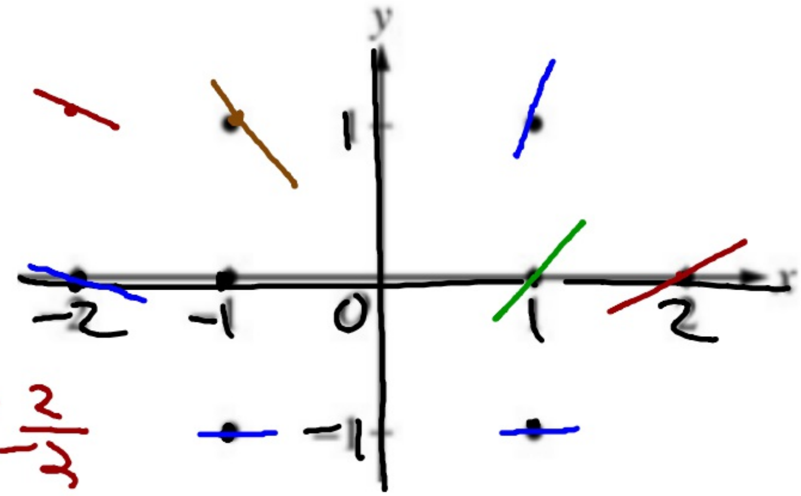
$\frac{dy}{dx} = e^x$

- Direct Variation $y = kx$
- Inverse Variation $y = \frac{k}{x}$

Slope Fields: A way to visualize solutions to differential equations

Very easy to make!!!!
Just plug coordinates of a point into dy/dx, simplify to a number,
ıen sketch a little line segment/gradient with slope equal to that number!

$$\frac{dy}{dx} = \frac{y+1}{x}$$



Construct the slope field where possible.

$(1,0) \to \frac{0+1}{1} \to 1$

$(2,0) \to \frac{1}{2} \to \frac{1}{2}$

$(0,0) \to \frac{1}{0} \to !!$

$(-1,1) \to \frac{2}{-1} \to -2$

$(-2,1) \to \frac{2}{-2} = -1$

Describe all points in the plane with a horizontal slope. $y = -1$