Solutions N. M.

D-DE1 Practice Assessment

1. An illness is spreading through a population of N people. Let R represent the number of people with the illness. The rate with respect to time of people with the illness is growing is directly proportional to the product of the number of people with the illness and the square root of the population size. Write a differential equation that models this situation.

N

$$\frac{dR}{dt} = kR \cdot \sqrt{N}$$

D-DE3:

2. Consider the differential equation $y' = 2y - 3$. Find the general solution y.

$$y' = \cancel{\frac{d}{dx}}\left(\frac{dy}{dx}\right) = (2y-3)\,dx$$

$$\frac{dy}{2y-3} = \frac{(2y-3)dx}{2y-3}$$

Sep. of Variables

$$\int \frac{1}{2y-3} \cdot dy = \int 1\, dx$$

Integrate

$$\frac{1}{2}\int 2 \cdot \frac{1}{2y-3}\, dy = x + C$$

$$\frac{1}{2} \ln|2y-3| + C = x + C$$

$$\frac{1}{2}\ln|2y-3| + \cancel{C} = x + C$$
$$\quad -\cancel{C} \qquad\qquad -C$$

could be different.

$$2\left(\frac{1}{2}\ln|2y-3|\right) = (x + C)2$$

$$\ln|2y-3| = 2x + C$$

Exponentiate

$x^n \cdot x^m = x^{n+m}$

$$e^{2x+C} = 2y-3$$

$$e^{2x} \cdot e^{C} = 2y - 3$$

$$\frac{Ce^{2x}}{2} + \frac{3}{2} = \frac{2y}{2}$$

$$Ce^{2x} + \frac{3}{2} = y$$

D-DE2: Consider the differential equation $\frac{dy}{dx} = 4y^2x$

3. Find the particular solution with initial condition (1,1/3)

$$\cancel{dx}\left(\frac{dy}{\cancel{dx}}\right) = (4y^2x)\,dx$$

$$\frac{dy}{y^2} = \frac{4\cancel{y^2}x\,dx}{\cancel{y^2}}$$

$$\int y^{-2}\,dy = \int 4x\,dx$$

$$\frac{y^{-1}}{-1} + C = 2x^2 + C$$

$$-\frac{1}{y} + \cancel{C} = 2x^2 + C$$
$$\quad\ \ -\cancel{C} \qquad\quad -C$$

$$-\frac{1}{y} = 2x^2 + C$$

$$-\frac{1}{y} = 2x^2 + C$$

← General Solution

before solving for y, let's find C

plug in $(1, \frac{1}{3})$

$$-\left(\frac{1}{\frac{1}{3}}\right) = 2(1)^2 + C$$

$$-3 = \cancel{2} + C$$
$$-2 \qquad -\cancel{2}$$

$$-5 = C$$

$$\cancel{-1}\left(-\frac{1}{y} = 2x^2 - 5\right)-1$$

$$\cancel{y}\left(\frac{1}{\cancel{y}}\right) = (5 - 2x^2)y$$

$$\frac{1}{5-2x^2} = \frac{\cancel{(5-2x^2)}y}{\cancel{5-2x^2}}$$

$$\boxed{\frac{1}{5-2x^2} = y}$$