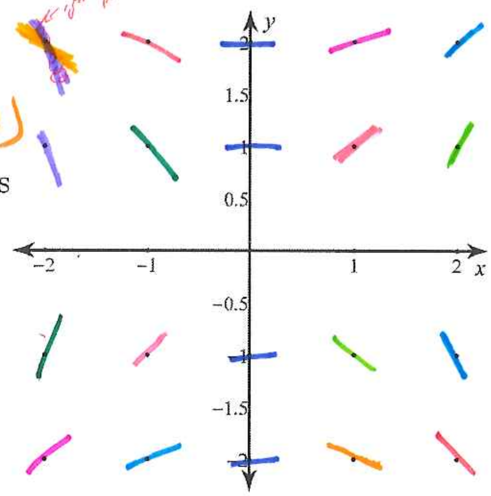
D-DE4 Q4 Practice Assessment #2

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y}$

1. On the axes provided, sketch a slope field at the points indicated (including the axes, where possible). $y = 0$ (x-axis is invalid)



2. While only some points are graphed, the slope field for #1 is defined for many others. Describe all points in the xy-plane that have positive slope.

$(1,1) \to \frac{1}{1}$

$(1,2) \to \frac{1}{2}$

$(2,1) \to \frac{2}{1}$

$(2,2) \to \frac{2}{2}$

$(0,1) \to \frac{0}{1} = 0$

$(0,2) \to \frac{0}{2} \to 0$

etc

$(-1,1) \to -\frac{1}{1}$

$(-1,2) \to -\frac{1}{2}$

$(-2,1) \to -2$

$(-2,2) \to -\frac{2}{2}$

$(-1,-1) \to 1$

$(-2,-2) \to 1$

$(-2,-1) \to 2$

$(-1,-2) \to \frac{1}{2}$

$(1,-1) \to -1$

$(2,-2) \to -1$

$(1,-2) \to -\frac{1}{2}$

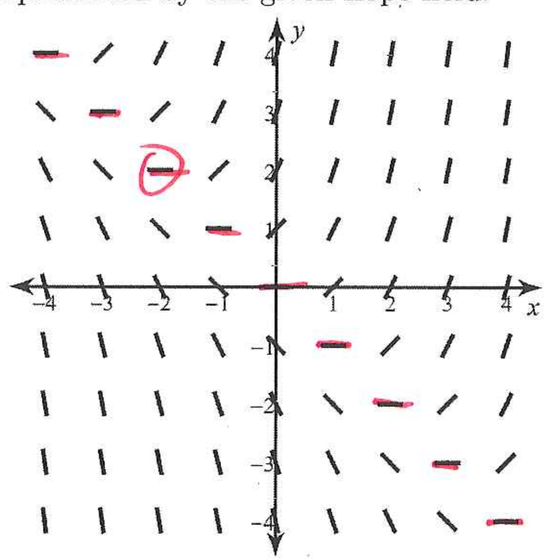
$(2,-1) \to -2$

$\frac{dy}{dx} = \frac{y}{x}$ is positive when the signs of x; y are the same. This is in the 1st and 3rd quadrants.

3. Choose the differential equation that could be represented by the given slope field.

~~$\frac{dy}{dx} = y - x$~~ $\quad$ $\frac{dy}{dx} = x + y$

~~$\frac{dy}{dx} = -xy$~~ $\quad$ ~~$\frac{dy}{dx} = xy$~~



Pick a test point like $(-2,2)$ $m = 0$

$y - x \to 2 - -2 \to 4 \neq 0$

$x + y \to -2 + 2 \to 0$

$-x \cdot y \to 2 \cdot 2 \to 4 \neq 0$

$xy \to -2 \cdot 2 \to -4 \neq 0$

Solutions N. M.

D-DE1 Practice Assessment

1. An illness is spreading through a population of N people. Let R represent the number of people with the illness. The rate with respect to time of people with the illness is growing is directly proportional to the product of the number of people with the illness and the square root of the population size. Write a differential equation that models this situation.

N

$$\frac{dR}{dt} = kR \cdot \sqrt{N}$$

D-DE3:

2. Consider the differential equation $y' = 2y - 3$. Find the general solution y.

$$y' = \cancel{d}\left(\frac{dy}{\cancel{dx}}\right) = (2y-3)\,dx$$

$$\frac{dy}{2y-3} = \frac{\cancel{(2y-3)}\,dx}{\cancel{2y-3}}$$

Sep. of Variables

$$\int \frac{1}{2y-3} \cdot dy = \int 1\,dx$$

Integrate

$$\frac{1}{2}\int 2 \cdot \frac{1}{2y-3}\,dy = x + C$$

$$\frac{1}{2} \ln|2y-3| + C = x + C$$

$$\frac{1}{2} \ln|2y-3| + \cancel{C} = x + C \quad -C \quad -C$$

could be different.

$$2\left(\frac{1}{2}\ln(2y-3)\right) = (x + C)\,2$$

$$\ln|2y-3| = 2x + C$$

Exponentiate

$$e^{2x+C} = 2y-3$$

$$x^n \cdot x^m = x^{n+m}$$

$$e^{2x} \cdot e^{C} = 2y-3$$

$$\frac{Ce^{2x}}{2} + \frac{3}{2} = \frac{2y}{2}$$

$$\boxed{Ce^{2x} + \frac{3}{2} = y}$$

D-DE2: Consider the differential equation $\frac{dy}{dx} = 4y^2x$

3. Find the particular solution with initial condition (1,1/3)

$$\cancel{dx}\left(\frac{dy}{\cancel{dx}}\right) = (4y^2x)\,dx$$

$$\frac{dy}{y^2} = \frac{4\cancel{y^2}x\,dx}{\cancel{y^2}}$$

$$\int y^{-2}\,dy = \int 4x\,dx$$

$$\frac{y^{-1}}{-1} + C = 2x^2 + C$$

$$-\frac{1}{y} + \cancel{C} = 2x^2 + C \quad -C$$

$$-\frac{1}{y} = 2x^2 + C$$

← General Solution

$$-\frac{1}{y} = 2x^2 + C$$

before solving for y, let's find C

plug in $(1, \frac{1}{3})$

$$-\left(\frac{1}{\frac{1}{3}}\right) = 2(1)^2 + C$$

$$-3 = 2 + C$$
$$-2 \quad\quad -2$$

$$-5 = C$$

$$\cancel{-1}\left(-\frac{1}{y} = 2x^2 - 5\right) \cdot -1$$

$$\cancel{y}\left(\frac{1}{\cancel{y}}\right) = (5 - 2x^2)\,y$$

$$\frac{1}{5-2x^2} = \frac{\cancel{(5-2x^2)}\,y}{\cancel{5-2x^2}}$$

$$\boxed{\frac{1}{5-2x^2} = y}$$