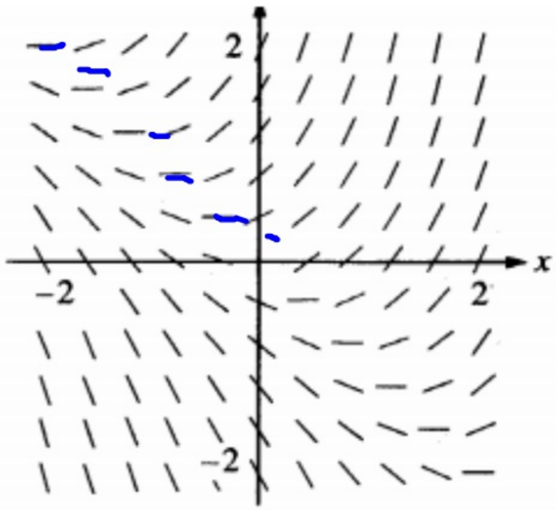
Good afternoon: attach warm up to notes

0 slopes $y = -x$

Shown is a slope field for which of the following differential equations?



(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$

$x - x = 0$

(D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

HW

use calcchat for odds

6 $r = 0.375\, s^2 + C$

12 $y = 11\sqrt{x^2 - 16} + C$

$$y = \frac{(\ln x)^2}{2} + C$$

18 Initial condition $(1, 2)$: $2 = C$

Particular solution: $y = \frac{1}{2}(\ln x)^2 + 2$

$$T - 70 = Ce^{-kt}$$

24 Initial condition:

$T(0) = 140$: $140 - 70 = 70 = Ce^0 = C$

Particular solution:

$T - 70 = 70e^{-kt}$, $T = 70\left(1 + e^{-kt}\right)$

30
$$m = \frac{dy}{dx} = \frac{y - 0}{x - 0} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\ln y = \ln x + C_1 = \ln x + \ln C = \ln Cx$$

$$y = Cx$$

36. (a) $\frac{dy}{dx} = ky^2$

(b) The direction field satisfies $(dy/dx) = 0$ along $y = 0$, and grows more positive as y increases. Matches (d).

Example: Let $\frac{dy}{dx} = \frac{-xy^2}{2}$ Find y with initial condition y(-1)=2

$$2\,dy = -xy^2\,dx$$

$$\frac{dy}{y^2} = -\frac{1}{2}x\,dx$$

$$\int y^{-2}\,dy = \int -\frac{1}{2}x\,dx$$

$$-y^{-1} = -\frac{1}{4}x^2 + C$$

$$y^{-1} = \frac{1}{4}x^2 + C$$

$$\frac{1}{y} = \frac{1}{4}x^2 + C$$

$$y = \frac{1}{\frac{1}{4}x^2 + C} \cdot \frac{4}{4} \Rightarrow y = \frac{4}{x^2 + C}$$

gen. sol.

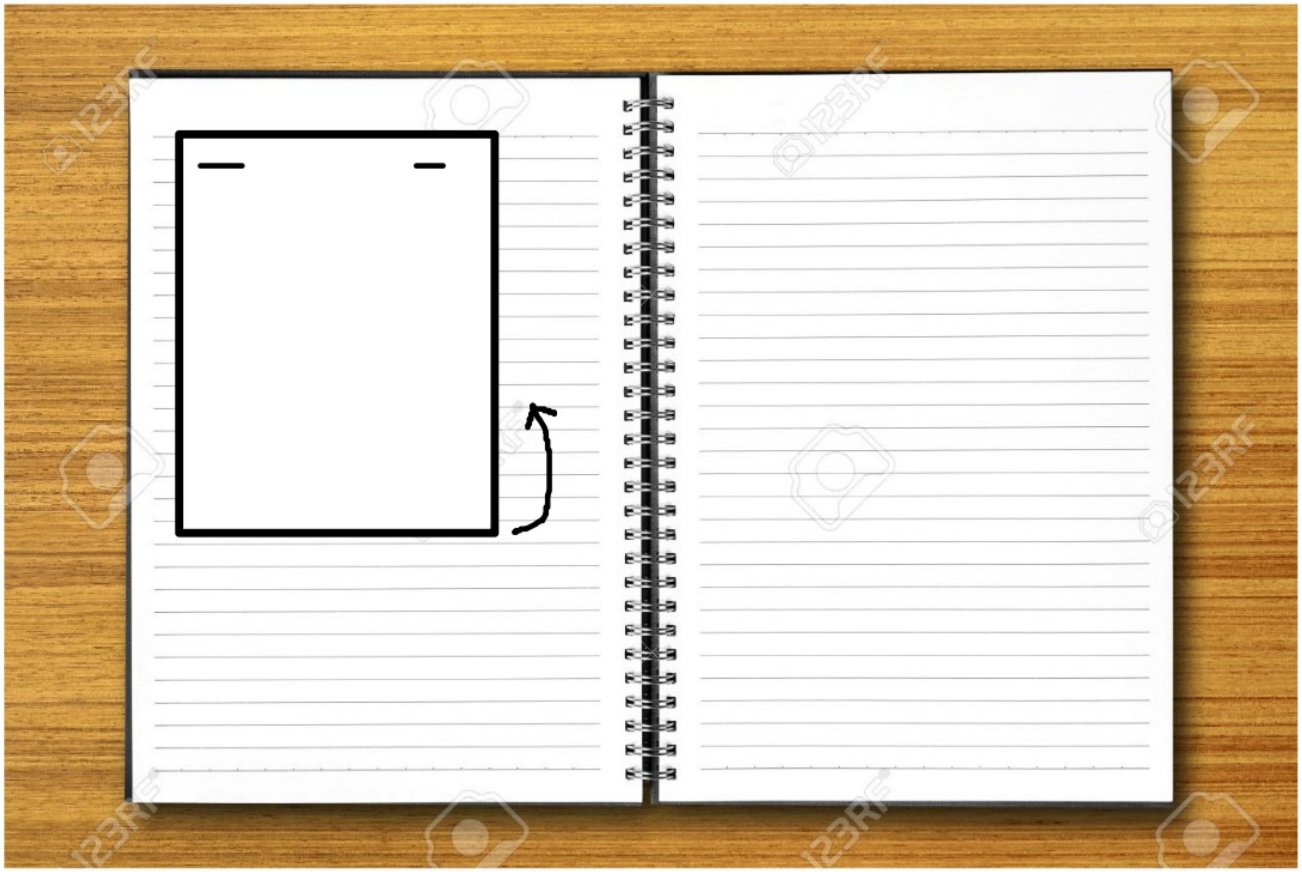
$$\boxed{y = \frac{4}{x^2 + 1}}$$

$(-1, 2) \rightarrow$

$$2 = \frac{4}{1 + C}$$

$$\frac{4}{2} = \frac{4}{1 + C} \rightarrow C = 1$$

Attach examples like this:



A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N - p)$

(C) $\frac{dp}{dt} = kp(p - N)$

(D) $\frac{dp}{dt} = kt(N - t)$

(E) $\frac{dp}{dt} = kt(t - N)$

$$\frac{dp}{dt} = k \cdot p \cdot (N - p)$$

heard rumor

haven't heard (everyone else)

Newton's Law of Cooling

The rate of change of the temperature of an object is proportional to the difference between its own temperature and that of its ambient environment.

k

$$\frac{dT}{dt} = -k(T - T_a)$$

Tomo works at a restaurant and prepares soup each evening before closing time to serve the next day. He wishes to place the soup pot in the refrigerator but the soup is too hot * immediately after cooking to adequately cool. The fridge cannot cool a pot that large if it is any warmer than 20C. To help speed up the cooling process to then use the refrigerator overnight, Tomo places the pot in a sink full of running cold water with a constant temperature of 5C. He finds that with occasional stirring, the soup would drop to 60C after 10 minutes in the water bath. How long before closing time should Tomo finish cooking the soup so he can cool it with the sink and keep it in the fridge overnight? * 100C

$T(0) = 100$ $T(t?) = 20$

$T(10) = 60$

$T_a = 5$

$\frac{dT}{dt} = -k(T - T_a)$

$\frac{dT}{dt} = -k(T-5)$

$\int \frac{dT}{T-5} = \int -k\, dt$

$\int 3\, dx$

$3x + C$

$\int \frac{1}{T-5} dT = -kt + C$

$\ln|T-5| = -kt + C$

$T - 5 = e^{-kt+C} \longrightarrow e^{-kt} \cdot e^{C}$

$T = Ce^{-kt} + 5 \longrightarrow T(t) = 95e^{-kt} + 5$

$(0,100)$

$100 = Ce^{0} + 5$

$95 = C$

$(10, 60)$

$60 = 95e^{-10k} + 5$

$55 = 95e^{-10k}$

$.579 = e^{-10k}$

$\ln(.579) = -10k$

$0.055 = k$

$T(t) = 95e^{-0.055t} + 5$

$(t, 20)$

$20 = 95e^{-0.055t} + 5$

$15 = 95e^{-0.055t} \longrightarrow t = 33.66$

If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

$$\frac{dy}{dx} = x^2 y \longrightarrow dy = x^2 y\, dx \rightarrow \int \frac{1}{y}\, dy = \int x^2\, dx$$

$$\ln|y| = \frac{1}{3}x^3 + C$$

$$y = e^{\frac{1}{3}x^3 + C}$$

$$\underline{y = Ce^{\frac{1}{3}x^3}}$$

2010AB6 nc

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

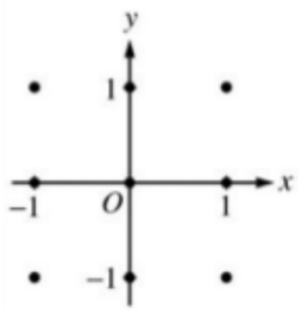
2006AB5b

nc

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

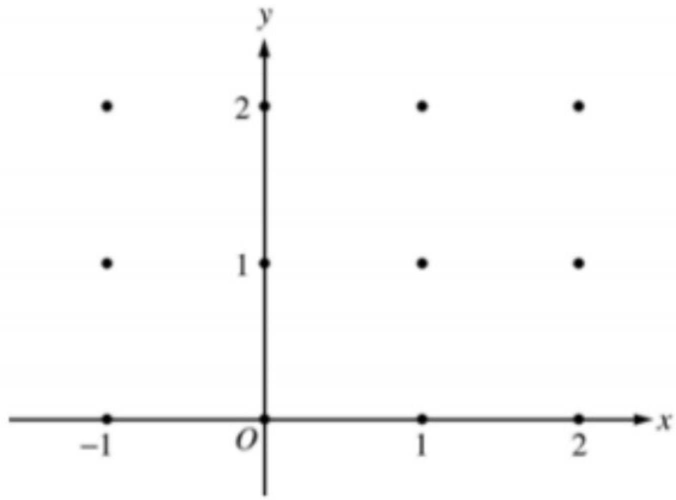
(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

2005AB6b

nc

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**



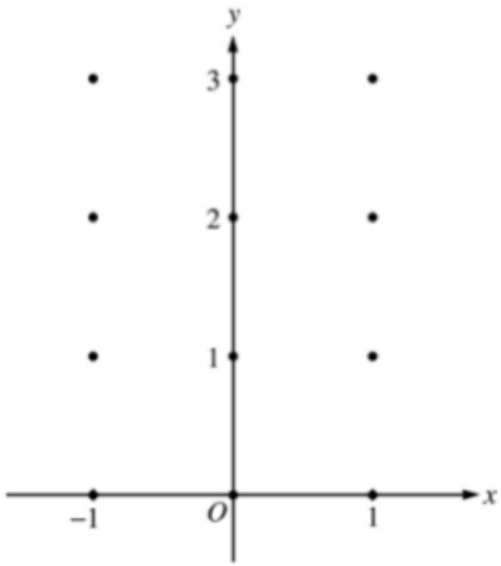
(b) Write an equation for the line tangent to the graph of f at $x = -1$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

2004AB6 nc

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the pink test booklet.)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.