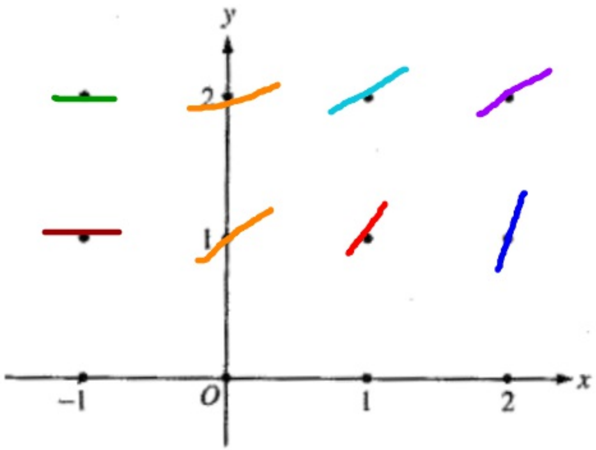
HW



2. wherever $y \neq 0$ and $x < 1$

3. C

4. $\frac{dF}{dt} = k(F)(F-R)$

5. $y = Ce^{3x} + 5/3$

6. $y = \sqrt{\frac{1}{\frac{17}{4} - 4x^3}}$

⑨ $y' = 3y - 5$

$$\frac{dy}{dx} = 3y - 5$$

$$dy = (3y - 5)\cdot dx$$

$$\int \frac{dy}{3y-5} = \int dx$$

$$\frac{1}{3}\int 3\,\frac{1}{3y-5}\cdot dy = x + C$$

$$\cancel{\tfrac{1}{3}}\ln|3y-5| = x + c\,)\,3$$

$$\ln|3y-5| = 3x + C$$

$$e^{3x+c} = 3y - 5$$

$$e^{3x}\cdot \cancel{e^{c}} = 3y - 5 \qquad C$$

$$Ce^{3x} + 5 = 3y \longrightarrow \boxed{y = Ce^{3x} + 5/3}$$

$\log_e x = a \rightleftarrows e^a = x$

(6) $\frac{dy}{dx} = 6y^3x^2$ $(1,2)$

$$\frac{dy}{y^3} = \frac{6\cancel{y^3}x^2 \cdot dx}{\cancel{y^3}}$$

$$\int y^{-3} \cdot dy = \int 6x^2 \cdot dx$$

$$\cancel{-2}\left(-\frac{1}{2}y^{-2} = 2x^3 + C\right)-2$$

$$\frac{1}{y^2} = C - 4x^3$$

$$y^2 = \frac{1}{C-4x^3} \rightarrow (1,2)$$

gen. sol. $\rightarrow y = \sqrt{\frac{1}{C-4x^3}}$

$$2 = \sqrt{\frac{1}{C-4(1)^3}}$$

$$4 = \frac{1}{C-4}$$

$$4C-16 = 1$$

$$4C = 17$$

$$C = 17/4$$

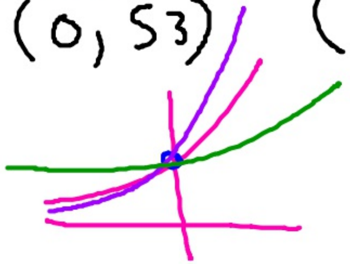
$$y = \sqrt{\frac{1}{\frac{17}{4} - 4x^3} \cdot \frac{4}{4}}$$

$$y = \sqrt{\frac{4}{17-16x^3}}$$

Applying differential equations

An infection is spreading at a rate proportional to the number infected. When the infection was discovered, there were 53 people sick. Four days later, 87 people are sick. How many people will have the infection after 10 days?

$(0, 53)$ $(4, 87)$



$$\frac{dy}{dt} = ky$$

!

$$y = Ce^{kt}$$

initial Value

!

$$y = 53e^{kt}$$

Find K

$$87 = 53e^{k \cdot 4}$$

$$\ln \frac{87}{53} = \cancel{\ln} e^{4k}$$

$$0.4956 = 4k$$

$$0.1239 = k$$

$$y = 53e^{0.1239t}$$

At $t = 10$

$$y = 53e^{0.1239 \times 10}$$

$$y \approx 182.97$$

$$y \approx 182 \text{ ppl.}$$

2. A bacteria culture containing 2,400 cells 3 hours ago has now grown to 5,200 cells. Assuming the rate of growth is proportional to the number present, determine the time (from now) at which the population will reach 10,000.

$y = Ce^{kt}$

$(0, 5200)$

$(-3, 2400)$ — t, y

$(?, 10{,}000)$

$y = 5200e^{kt}$

$2400 = 5200e^{-3k}$

ln $0.462 = e^{-3k}$

$-0.773 = -3k$

$0.2577 = k$

$y = 5200e^{0.2577t}$

$10{,}000 = 5200e^{0.2577t}$

$1.923 = e^{.2577t}$

ln $.6539 = .2577t$

$2.538 = t$

Newton's Law of Cooling:

$$\frac{dT}{dt} = k(T-S)$$

where S is the surrounding temperature

Suppose an object that is 1200 C is put into an environment with a constant 80 C temperature. After an hour, the object's temperature is 950 C. Find the temperature after 5 hours in the environment.

$$\frac{dT}{\cancel{dt}} = k(T-80)\,dt$$

$$\frac{dT}{T-80} = k\,dt$$

$$\int \frac{1}{T-80}\,dT = \int k\,dt$$

$$\ln|T-80| = kt + C$$

$$Ce^{kt} = T-80$$

$$Ce^{kt} + 80 = T$$

$$1120e^{kt} + 80 = T$$

$(1, 950)$

$$1120e^{k\cdot 1} + 80 = 950$$

$$e^{k} = .7767$$

$$k = \ln(.7767)$$

$$k \approx -0.253$$

$$1120e^{-0.253(5)} + 80 = T$$

$$\boxed{396.75C = T}$$

Carbon-14 is a radioactive isotope that decays at a rate proportional to the amount of matter present.
It has a half-life of 5600 years. What fraction of the original amount of Carbon-14 in a sample would be present after 10,000 years?

Let M = mass of carbon

$$M = Ce^{kt}$$

After 5600 years, $M = \frac{1}{2}C$ if C is initial amount.

So: $\left(5600, \frac{1}{2}C\right)$

Subbing in

$$\frac{1}{2}C = Ce^{k \cdot 5600}$$

$$\frac{1}{2} = e^{k \cdot 5600}$$

ln

$$\ln\frac{1}{2} = k \cdot 5600$$

$$\frac{\ln\frac{1}{2}}{5600} = k$$

$$0.0001238 \approx k$$

$$M = Ce^{0.0001238 \cdot t}$$

At $t = 10{,}000$ years:

$$M = Ce^{(0.0001238)(10{,}000)}$$

$$M = C\left(e^{-1.238}\right)$$

$$M = C(0.2900)$$

30% left

AP Testers: work on the packet of multiple choice Q's

Woo haa!

Non-testers: begin planning the roller coaster project

Due: 5/6

Practice test will be handed out at 3:50p